CANADIAN BOARD OF EXAMINERS FOR PROFESSIONAL SURVEYORS

C2 - LEAST SQUARES ESTIMATION & DATA ANALYSIS October 2016

Although programmable calculators may be used, candidates must show all formulae used, the substitution of values into them, and any intermediate values to 2 more significant figures than warranted for the answer. Otherwise, full marks may not be awarded even though the answer is numerically correct.

Note: This examination consists of 8 questions on 3 pages. **Marks**

Time: 3 hours

Q. No		Value	Earned
1.	Explain the difference of the following terms: a) Precision and Accuracy b) Standard Deviation and Root Mean Square Error c) Covariance and Correlation Coefficient d) Redundancy of a linear system and redundancy number e) Type I and Type II Errors in statistical testing	15	
2.	Given the cofactor matrix Q of the horizontal coordinates (x, y) of a survey station and the unit variance $\sigma_0^2 = 2\,cm^2$, calculate the semi-major, semi-minor axis and the orientation of the standard error ellipse associated with this station. $$Q = \begin{bmatrix} 5.32 & 6.02 \\ 6.02 & 8.38 \end{bmatrix}$$	10	
3.	Given the following mathematical model $f(\ell, x) = 0 \quad C_\ell \quad C_x$ where f is the vector of mathematical models, x is the vector of unknown parameters and C_x is its variance matrix, ℓ is the vector of observations and C_ℓ is its variance matrix. a) Linearize the mathematical model b) Formulate the variation function c) Derive the least squares normal equation d) Derive the least squares solution of the unknown parameters	15	
4.	Given the variance-covariance matrix of the measurement vector $\ell = \begin{bmatrix} \ell_1 \\ \ell_2 \end{bmatrix}$: $$C_\ell = \begin{bmatrix} \frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} \end{bmatrix}$$ and two functions of ℓ : $x = \ell_1 + \ell_2$ and $y = 3\ell_1$, determine $C_{xy}, C_{x\ell}, C_{y\ell}$	10	

5.

The distance between Point A and Point B has been independently measured 5 times with different precisions. The distance measurements and their weights are given in the following table. Determine the weighted mean of the distance and its precision.

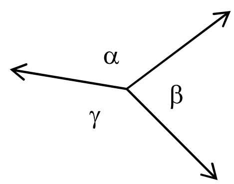
	Measurements (m)	**Weights**
1	100.005	1
2	100.001	3
3	99.997	2
4	100.002	2
5	99.996	1

10

6.

Given the angle measurements at a station along with their standard deviations:

Angle	**Measurement**	**Standard Deviation**
α	104°38'56"	6.7"
β	113°17'35"	9.9"
γ	142°03'14"	4.3"



Perform least squares adjustment to the problem using

a) Conditional equations (conditional adjustment)
b) Observation equations (parametric adjustment)

25

7.

Given the sample unit variance obtained from the adjustment of a geodetic network $\hat{\sigma}_0^2 = 0.55\,\text{cm}^2$ with a degree of freedom $\upsilon = 3$ and the a-priori standard deviation $\sigma_0 = 0.44\,\text{cm}$, conduct a statistic test to decide if the adjustment result is acceptable with a significance level of $\alpha = 5\%$.

The critical values that might be required in the testing are provided in the following table:

α	0.001	0.01	0.025	0.05	0.10
$\chi^2_{\alpha,\ \upsilon=3}$	16.26	11.34	9.35	7.82	6.25

where $\chi^2_{\alpha,\ \upsilon=3}$ is determined by the equation $\alpha = \int_{\chi^2_{\alpha,\ \upsilon=3}}^{\infty} \chi^2(x)dx$.

5

8. The following residual vector $\hat{r}$ and estimated cofactor matrix $Q_{\hat{r}}$ were computed from a least squares adjustment using independent observations with a standard deviation of $\sigma_0 = 1.5$ mm:

$$\hat{r} = \begin{bmatrix} 4 & 2 & -3 & 10 \end{bmatrix} \qquad \text{(mm)}$$

$$Q_{\hat{r}} = \begin{bmatrix} 15 & 1 & 3 & -2 \\ 1 & 7 & -1 & 3 \\ 3 & -1 & 4 & -1 \\ -2 & 3 & -1 & 2 \end{bmatrix} \qquad (\text{mm}^2)$$

Given that a global test has been rejected with a significance level of $\alpha = 0.04$, conduct further tests to identify which observation(s) may contain an outlier. The critical values that might be required in the testing are provided in the following table:

α	0.001	0.002	0.003	0.004	0.005	0.01	0.05
K_α	3.09	2.88	2.75	2.65	2.58	2.33	1.64

where K_α is determined by the equation $\alpha = \int_{K_\alpha}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-x^2/2} dx$.

10

Total Marks: 100