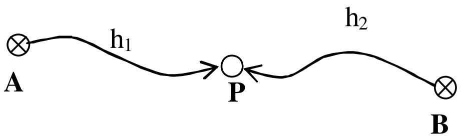
CANADIAN BOARD OF EXAMINERS FOR PROFESSIONAL SURVEYORS

C-2 LEAST SQUARES ESTIMATION & DATA ANALYSIS

October 2013

Although programmable calculators may be used, candidates must show all formulae used, the substitution of values into them, and any intermediate values to 2 more significant figures than warranted for the answer. Otherwise, full marks may not be awarded even though the answer is numerically correct.

Note: This examination consists of 8 questions on 3 pages. **Marks**

Q. No	Time: 3 hours	Value	Earned
1.	Define and explain briefly the following: a) Standard deviation b) Covariance c) Correlation coefficient d) Type II error e) Root mean square error f) Internal reliability	2.5 2.5 2.5 2.5 2.5 2.5	
2.	Given the cofactor matrix Q of the horizontal coordinate (x, y) of a survey station and the unit variance $\hat{\sigma}_0^2 = 2\,\text{cm}^2$, calculate the semi-major, semi-minor axis, and the orientation of the standard error ellipse associated with this station. $Q = \begin{bmatrix} 5.32 & 6.02 \\ 6.02 & 8.38 \end{bmatrix}$	10	
3.	Given a leveling network below where A and B are known points, h_1 and h_2 are two height difference measurements with standard deviation of σ_1 and σ_2, respectively and $\sigma_1 = 2\ \sigma_2$. Determine the value of σ_1 and σ_2 so that the standard deviation of the height solution at point P using least squares adjustment is equal to 2mm. 	10	
4.	Given the variance-covariance matrix of the measurement vector $\ell = \begin{bmatrix} \ell_1 \\ \ell_2 \end{bmatrix}$: $C_\ell = \begin{bmatrix} \frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} \end{bmatrix}$ and two functions of $\ell : x = \ell_1 + \ell_2$ and $y = 3\ell_1$, determine $C_{xy}, C_{x\ell}, C_{y\ell}$	10	

5\.

Given the following mathematical model

$$f(\ell, x) = 0 \qquad C_\ell \quad C_x$$

where f is the vector of mathematical models, x is the vector of unknown parameters and C_x is its variance matrix, ℓ is the vector of observations and C_ℓ is its variance matrix.

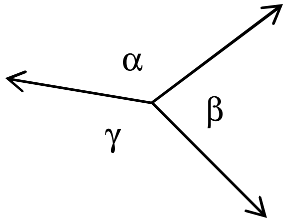
a) Linearize the mathematical model 5

b) Formulate the variation function 5

6\.

Given the angle measurements at a station along with their standard deviations:

Angle	Measurement	Standard Deviation
α	134°38’56”	6.7”
β	83°17’35”	9.9”
γ	142°03’14”	4.3”



Perform least squares adjustment to the problem using

a) Conditional equations (conditional adjustment) 12.5

b) Observation equations (parametric adjustment) 12.5

7\.

Given the sample unit variance obtained from the adjustment of a geodetic network $\hat{\sigma}_0^2 = 0.55\,\text{cm}^2$ with a degree of freedom $\upsilon = 3$ and the a-priori standard deviation $\sigma_0 = 0.44\,\text{cm}$, conduct a statistical test to decide if the adjustment result is acceptable with a significance level of $\alpha = 5\%$.

The critical values that might be required in the testing are provided in the following table: 10

α	0.001	0.01	0.025	0.05	0.10
$\chi^2_{\alpha,\ \upsilon=3}$	16.26	11.34	9.35	7.82	6.25

where $\chi^2_{\alpha,\ \upsilon=3}$ is determined by the equation $\alpha = \int_{\chi^2_{\alpha,\ \upsilon=3}}^{\infty} \chi^2(x)dx$.

8.

The following residual vector $\hat{r}$ and estimated cofactor matrix $Q_{\hat{r}}$ were computed from a least squares adjustment using independent observations with a standard deviation of $\sigma_0 = 1.5$ mm:

$$\hat{r} = \begin{bmatrix} 4 & 2 & -3 & 10 \end{bmatrix} \qquad \text{(mm)}$$

$$Q_{\hat{r}} = \begin{bmatrix} 15 & 1 & 3 & -2 \\ 1 & 7 & -1 & 3 \\ 3 & -1 & 4 & -1 \\ -2 & 3 & -1 & 2 \end{bmatrix} \qquad (\text{mm}^2)$$

Given that a global test has been rejected with a significance level of $\alpha = 0.04$, conduct further tests to identify which observation(s) may contain an outlier. The critical values that might be required in the testing are provided in the following table:

α	0.001	0.002	0.003	0.004	0.005	0.01	0.05
K_α	3.09	2.88	2.75	2.65	2.58	2.33	1.64

where K_α is determined by the equation $\alpha = \int_{K_\alpha}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-x^2/2} dx$.

10

Total Marks: 100