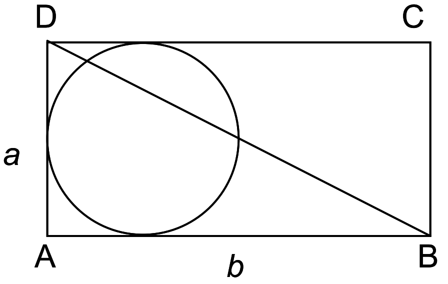
CANADIAN BOARD OF EXAMINERS FOR PROFESSIONAL SURVEYORS

C-2 LEAST SQUARES ESTIMATION & DATA ANALYSIS

March 2017

Note: This examination consists of 8 questions on 3 pages.

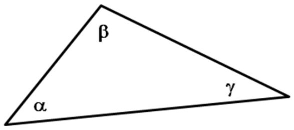
Time: 3 hours

Q. No		Marks Value	Marks Earned
1.	Define or explain briefly the following terms: a) Precision b) Accuracy c) Redundancy of a linear system d) Correlation coefficient e) Internal reliability	10	
2.	Sides *a* and *b* are measured once each as follows: $\boldsymbol{l} = \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 10 \\ 20 \end{bmatrix} \text{m}$ $\boldsymbol{C}_l = \begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix} \text{cm}^2$  a) Estimate the areas of triangle ABD and the circle shown inside the rectangle. b) Estimate the standard deviations of the quantities computed in Part a). c) Estimate the correlation between the triangle and the circle estimates. d) Discuss the nature of the correlations computed in Part c).	15	
3.	Consider that the shape of an object is defined by the following equation: $z_i = ax_i^3 + b\sin(y_i)$, i = 1, 2, 3. where z_i, x_i, y_i are observations with standard deviations $\sigma_{z_i}, \sigma_{x_i}, \sigma_{y_i}$, and *a* and *b* are parameters to be estimated. Derive the linearized form of this non-linear model for least squares adjustment including the required matrices and vectors.	10	
4.	Given the variance-covariance matrix of the horizontal coordinates (x, y) of a survey station, determine the semi-major, semi-minor axis and the orientation of the standard error ellipse associated with this station. $C_x = \begin{bmatrix} 0.000532 & 0.000602 \\ 0.000602 & 0.000838 \end{bmatrix} \text{m}^2$	10	

5. Prove that $\frac{\sigma}{\sqrt{n}}$ is the standard deviation of the mean value $\bar{x} = \frac{\sum_{i=1}^{n} \ell_i}{n}$, each measurement ℓ_i is made with a standard deviation σ. 10

6. Given the angle measurements of a triangle along with their standard deviations:

Angle	Measurement	Standard Deviation
α	104°38’56”	6.7”
β	33°17’35”	9.9”
γ	42°03’14”	4.3”



Perform least squares adjustment to the problem using

a) Conditional equations (conditional adjustment) 12.5
b) Observation equations (parametric adjustment) 12.5

7. A baseline of calibrated length (μ) 200.0m is measured 5 times. Each measurement is independent and made with the same precision. The sample mean ($\bar{x}$) and sample standard deviation (s) are calculated from the measurements:

$\bar{x}$ = 200.5m s = 0.05m

Test at the 95% level of confidence if the measured distance is significantly different from the calibrated distance.

The critical value that might be required in the testing is provided in the following table: 10

	t_α			
Degree of freedom	$t_{0.90}$	$t_{0.95}$	$t_{0.975}$	$t_{0.99}$
1	3.08	6.31	12.7	31.8
2	1.89	2.92	4.30	6.96
3	1.64	2.35	3.18	4.54
4	1.53	2.13	2.78	3.75
5	1.48	2.01	2.57	3.36

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<p>Given the sample unit variance obtained from the adjustment of a geodetic network $\hat{\sigma}_0^2 = 0.55\, cm^2$ with a degree of freedom $\upsilon = 3$ and the a-priori standard deviation $\sigma_0 = 0.44\,\text{cm}$, conduct a statistic test to decide if the adjustment result is acceptable with a significance level of $\alpha = 5\%$. Provide the major test steps and explain the conclusion.</p>
<p>The critical values that might be required in the testing are provided in the following table:</p>
<table>
<tr><td>α</td><td>0.001</td><td>0.01</td><td>0.025</td><td>0.05</td><td>0.10</td></tr>
<tr><td>$\chi^2_{\alpha,\ \upsilon=3}$</td><td>16.26</td><td>11.34</td><td>9.35</td><td>7.82</td><td>6.25</td></tr>
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