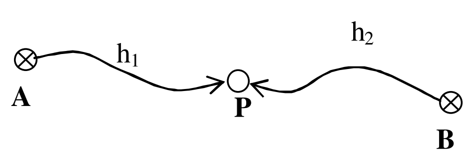
CANADIAN BOARD OF EXAMINERS FOR PROFESSIONAL SURVEYORS

C2 - LEAST SQUARES ESTIMATION & DATA ANALYSIS **March 2015**

Although programmable calculators may be used, candidates must show all formulae used, the substitution of values into them, and any intermediate values to 2 more significant figures than warranted for the answer. Otherwise, full marks may not be awarded even though the answer is numerically correct.

Note: This examination consists of 9 questions on 3 pages.

Time: 3 hours

Q. No		Marks Value	Marks Earned
1.	Briefly explain the following terms: a) Precision and accuracy b) Type I and Type II errors in statistical testing c) Statistically independent and uncorrelated d) Standard deviation and root mean square error	10	
2.	Given the following mathematical model $f(\ell, x) = 0 \quad C_\ell \quad C_x$ where f is the vector of mathematical models, x is the vector of unknown parameters and C_x is its variance matrix, ℓ is the vector of observations and C_ℓ is its variance matrix a) Derive the least squares normal equation. b) Derive the least squares solution of the unknown parameters and their variance-covariance matrix.	15	
3.	Given a leveling network below where A and B are known points, h_1 and h_2 are two height difference measurements with standard deviation of σ_1 and σ_2, respectively and $\sigma_1 = 2\,\sigma_2$. Determine the value of σ_1 and σ_2 so that the standard deviation of the height solution at P using least squares adjustment is equal to 2mm. 	10	

<table>
<tr>
<td>4.</td>
<td>Given the variance-covariance matrix of the measurement vector $\ell = \begin{bmatrix} \ell_1 \\ \ell_2 \end{bmatrix}$:

$$C_\ell = \begin{bmatrix} \frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} \end{bmatrix}$$
and the function $x = \ell_1 + \ell_2$, determine C_x.</td>
<td>5</td>
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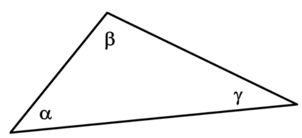
<td>5.</td>
<td>Given the angle measurements at a station along with their standard deviations, conduct a conditional least squares adjustment. You are required to compute the following quantities:
a) the estimated residuals
b) the variance-covariance matrix of the estimated residuals
c) the estimated observations
d) the variance-covariance matrix of the estimated observations
e) the estimated variance factor.

<table>
<tr><th>Angle</th><th>Measurement</th><th>Standard Deviation</th></tr>
<tr><td>α</td><td>104°38’56”</td><td>6.7”</td></tr>
<tr><td>β</td><td>33°17’35”</td><td>9.9”</td></tr>
<tr><td>γ</td><td>42°03’14”</td><td>4.3”</td></tr>
</table>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<td>6.</td>
<td>Conduct a parametric least squares adjustment to the same data given in Problem 5. You are required to compute the following quantities:
a) the estimated parameters
b) the variance-covariance matrix of the estimated parameters
c) the estimated difference between α and β
d) the variance of the estimated difference between α and β.</td>
<td>10</td>
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<td>7.</td>
<td>Given the variance-covariance matrix of the horizontal coordinates (x, y) of a survey station, determine the semi-major, semi-minor axis and the orientation of the standard error ellipse associated with this station.

$$C_x = \begin{bmatrix} 0.0484 & 0.0246 \\ 0.0246 & 0.0196 \end{bmatrix} \ m^2$$</td>
<td>10</td>
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8.

A distance has been independently measured 4 times and its sample unit variance obtained from the adjustment $\hat{\sigma}_0^2$ is equal to 1.44 cm. If the a-priori standard deviation σ_0 is 1.0 cm, conduct a statistic test to decide if the adjustment result is acceptable with a significance level of $\alpha = 5\%$. Show all work including the formulas used and explain all steps. The critical values that might be required in the testing are provided in the following table:

α	0.001	0.01	0.025	0.05	0.10
$\chi^2_{\alpha,\ \upsilon=3}$	16.26	11.34	9.35	7.82	6.25

where $\chi^2_{\alpha,\ \upsilon=3}$ is determined by the equation $\alpha = \int_{\chi^2_{\alpha,\ \upsilon=3}}^{\infty} \chi^2(x)dx$ and υ is the degree of freedom.

10

9.

An angle has been measured independently 5 times with the same precision and the observed values are given in the following table. Test at the 95% level of confidence if the sample mean is significantly different from the true angle value $45^o00'00''$. Show all work including the formulas used and explain all steps.

α_1	α_2	α_3	α_4	α_5
$45^o00'05''$	$45^o00'10''$	$44^o59'58''$	$45^o00'07''$	$44^o59'54''$

The critical value that might be required in the testing is provided in the following table:

	t_α			
Degree of freedom	$t_{0.90}$	$t_{0.95}$	$t_{0.975}$	$t_{0.99}$
1	3.08	6.31	12.7	31.8
2	1.89	2.92	4.30	6.96
3	1.64	2.35	3.18	4.54
4	1.53	2.13	2.78	3.75
5	1.48	2.01	2.57	3.36

15

Total Marks: 100