CANADIAN BOARD OF EXAMINERS FOR PROFESSIONAL SURVEYORS

C-2 LEAST SQUARES ESTIMATION & DATA ANALYSIS **March 2013**

Although programmable calculators may be used, candidates must show all formulae used, the substitution of values into them, and any intermediate values to 2 more significant figures than warranted for the answer. Otherwise, full marks may not be awarded even though the answer is numerically correct.

Note: This examination consists of 8 questions on 3 pages. **Marks**

Q. No — Time: 3 hours — Value — Earned

1. Explain briefly the difference between:

a) Precision and accuracy. — 2.5
b) Type I and Type II errors in statistical testing. — 2.5
c) Statistically independent and uncorrelated. — 2.5
d) Standard deviation and root mean square error. — 2.5

2. Given the following mathematical model

$$f(\ell, x) = 0 \quad C_\ell \quad C_x$$

where f is the vector of mathematical models, x is the vector of unknown parameters and C_x is its variance matrix, ℓ is the vector of observations and C_ℓ is its variance matrix.

a) Linearize the mathematical model. — 5
b) Formulate the variation function. — 5
c) Derive the least squares solution of the unknown parameters. — 10

3. The distance between Point A and Point B has been independently measured 5 times with the same precision using a distance measuring device and the standard deviation of the obtained mean distance is 1.58cm. Determine the precision of the distance measurement. — 5

4. Given the angle measurements of a triangle along with their standard deviations:

Angle	Measurement	Standard Deviation
α	104°38'56"	6.7"
β	33°17'35"	9.9"
γ	42°03'14"	4.3"

Perform least squares adjustment to the problem using

a) Conditional equations (conditional adjustment). — 12.5
b) Observation equations (parametric adjustment). — 12.5

5. Given the variance-covariance matrix of the horizontal coordinates (x, y) of a survey station, determine the semi-major, semi-minor axis and the orientation of the standard error ellipse associated with this station. **10**

$$C_x = \begin{bmatrix} 0.000532 & 0.000602 \\ 0.000602 & 0.000838 \end{bmatrix} \ m^2$$

6. A distance has been independently measured 4 times and its sample unit variance obtained from the adjustment $\hat{\sigma}_0^2$ is equal to 1.44 cm. If the a-priori standard deviation σ_0 is 1.0 cm, conduct a statistic test to decide if the adjustment result is acceptable with a significance level of $\alpha = 5\%$. The critical values that might be required in the testing are provided in the following table: **10**

α	0.001	0.01	0.025	0.05	0.10
$\chi^2_{\alpha,\ \upsilon=3}$	16.26	11.34	9.35	7.82	6.25

where $\chi^2_{\alpha,\ \upsilon=3}$ is determined by the equation $\alpha = \int_{\chi^2_{\alpha,\ \upsilon=3}}^{\infty} \chi^2(x)dx$ and υ is the degree of freedom.

7. An angle has been measured independently 5 times with the same precision and the observed values are given in the following table. Test at the 95% level of confidence if the sample mean is significantly different from the true angle value 45°00’00”. **15**

α_1	α_2	α_3	α_4	α_5
45°00’05”	45°00’10”	44°59’58”	45°00’07”	44°59’54”

The critical value that might be required in the testing is provided in the following table:

	t_α			
Degree of freedom	$t_{0.90}$	$t_{0.95}$	$t_{0.975}$	$t_{0.99}$
1	3.08	6.31	12.7	31.8
2	1.89	2.92	4.30	6.96
3	1.64	2.35	3.18	4.54
4	1.53	2.13	2.78	3.75
5	1.48	2.01	2.57	3.36

8.	Given the variance-covariance matrix of the measurement vector $\ell = \begin{bmatrix} \ell_1 \\ \ell_2 \end{bmatrix}$: $C_\ell = \begin{bmatrix} \frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} \end{bmatrix}$ and the function $x = \ell_1 + \ell_2$, determine C_x.	5	
	Total Marks:	100	