ASSOCIATION OF CANADA LANDS SURVEYORS - BOARD OF EXAMINERS
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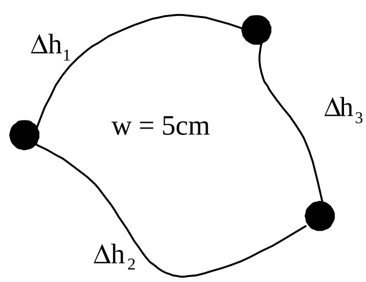
SCHEDULE I / ITEM 2
LEAST SQUARES ESTIMATION AND DATA ANALYSIS

February 2001
(1990 Regulations)
This examination consists of __5__ questions on __2__ pages **(Closed Book)**

Q. No.	**Time: 3 hours**	**Marks**
1.	Perform a least squares adjustment of the following leveling network in which three height differences Δh_i, i = 1, 2, 3 were observed.  The misclosure w is 5cm. Each Δh_i was measured with a variance of 2 cm^2.	25
2.	Given the following direct model for x and y as a function of ℓ_1, ℓ_2 and ℓ_3: $$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 & -2 & 1 \\ 5 & 3 & -1 \end{bmatrix} \begin{bmatrix} \ell_1 \\ \ell_2 \\ \ell_3 \end{bmatrix}$$ where the covariance matrix of the ℓ's is $C_\ell = \begin{bmatrix} 4 & -2 & -1 \\ -2 & 2 & 1 \\ -1 & 1 & 2 \end{bmatrix}$ Compute the covariance matrix for x and y.	15
3.	Define and explain briefly the following terms: a) Statistical hypothesis b) Degree of freedom of a linear system c) Accuracy d) Design matrix e) Precision	10

4.

Given the following mathematical models

$$\mathbf{f}_1(\ell_1, \mathbf{x}_1) = 0 \qquad \mathbf{C}_{\ell_1} \quad \mathbf{C}_{x_1}$$
$$\mathbf{f}_2(\ell_2, \mathbf{x}_1, \mathbf{x}_2) = 0 \qquad \mathbf{C}_{\ell_2} \quad \mathbf{C}_{x_2}$$

where f_1 and f_2 are vectors of mathematical models, x_1 and x_2 are vectors of unknown parameter, ℓ_1 and ℓ_2 are vectors of observations, $\mathrm{C}_{\ell_1}, \mathrm{C}_{\ell_2}, \mathrm{C}_{x_1}$ and C_{x_2} are covariance matrices.

a) Formulate the variation function.

b) Derive the most expanded form of the least squares normal equation system.

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5.

The following residual vector $\hat{\mathrm{r}}$ and estimated covariance matrix $\mathrm{C}_{\hat{\mathrm{r}}}$ were computed from a least squares adjustment using five independent observations with a standard deviation of $s = 2$mm and a degree of freedom $u = 2$:

$$\hat{\mathrm{r}} = \begin{bmatrix} 4 & 2 & -3 & 9 & -5 \end{bmatrix} \qquad \text{(mm)}$$

$$\mathrm{C}_{\hat{\mathrm{r}}} = \begin{bmatrix} 9 & -1 & 2 & -3 & 5 \\ -1 & 1 & 1 & 3 & -2 \\ 2 & 1 & 4 & -2 & 3 \\ -3 & 3 & -2 & 4 & -1 \\ 5 & -2 & 3 & -1 & 16 \end{bmatrix} \qquad (\text{mm}^2)$$

Given α = 0.01,

a) Conduct a global test to decide if there exists outlier or not.

b) If the test in a) fails, conduct local tests to locate the outlier.

The critical values that might be required in the testing are provided in the following tables:

α	0.001	0.01	0.02	0.05	0.10
$c^2_{a,\ u=2}$	13.82	9.21	7.82	5.99	4.61

α	0.001	0.002	0.003	0.004	0.005	0.01	0.05
K_α	3.09	2.88	2.75	2.65	2.58	2.33	1.64

where $c^2_{a,\ u=2}$ is determined by the equation $a = \int_{c^2_{a,\ u=2}}^{\infty} c^2(x)dx$ and K_α is determined by the equation $\alpha = \int_{K_a}^{\infty} \frac{1}{\sqrt{2p}} e^{-x^2/2} dx$.

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Total Marks: 100