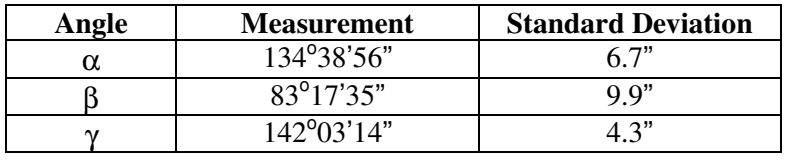
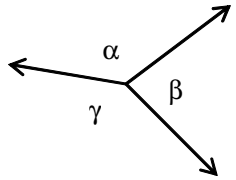
ASSOCIATION OF CANADA LANDS SURVEYORS - BOARD OF EXAMINERS
WESTERN CANADIAN BOARD OF EXAMINERS FOR LAND SURVEYORS
ATLANTIC PROVINCES BOARD OF EXAMINERS FOR LAND SURVEYORS

SCHEDULE I / ITEM 2 **March 2005**

LEAST SQUARES ESTIMATION AND DATA ANALYSIS

Note: This examination consists of _6_ questions on _2_ pages. **Marks**

Time: 3 hours

Q. No		Value	Earned
1	Define and briefly explain the following terms a) Precision b) Accuracy c) Correlation coefficient d) Redundancy of a linear system e) Unbiasedness of an estimator	10	
2	Given the following mathematical model $f(\ell, x) = 0 \quad C_\ell \quad C_x$ where f is the vector of mathematical model, x is the vector of unknown parameter and C_x is its variance matrix, ℓ is the vector of observations and C_ℓ is its variance matrix. a) Formulate the variation function. b) Derive the least squares normal equation. c) Derive the least squares solution of the unknown parameters.	15	
3	Given the angle measurements at a station along with their standard deviations:   Perform least squares adjustment to the problem using a) Conditional equations (conditional adjustment) b) Observation equations (parametric adjustment)	30	

<table>
<tr>
<td>4</td>
<td>Given the variance-covariance matrix of the horizontal coordinates (x, y) of a survey station, determine the semi-major and semi-minor axes and the orientation of the standard error ellipse associated with this station.
$$C_x = \begin{bmatrix} 0.000532 & 0.000602 \\ 0.000602 & 0.000838 \end{bmatrix} \ m^2$$</td>
<td>10</td>
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<td>5</td>
<td>A baseline of calibrated length (μ) 100.0m is measured 5 times. Each measurement is independent and made with the same precision. The sample mean ($\bar{x}$) and sample standard deviation (s) are calculated from the measurements:
$\bar{x}$ =100.5m s = 0.05m
a) Describe the major steps to test the mean value.
b) Test at the 5% level of confidence if the measured distance is significantly different from the calibrated distance.
The critical value that might be required in the testing is provided in the following table:
Percentiles of t distribution
<table>
<tr><td></td><td colspan="4">t_α</td></tr>
<tr><td>Degree of freedom</td><td>$t_{0.90}$</td><td>$t_{0.95}$</td><td>$t_{0.975}$</td><td>$t_{0.99}$</td></tr>
<tr><td>1</td><td>3.08</td><td>6.31</td><td>12.7</td><td>31.8</td></tr>
<tr><td>2</td><td>1.89</td><td>2.92</td><td>4.30</td><td>6.96</td></tr>
<tr><td>3</td><td>1.64</td><td>2.35</td><td>3.18</td><td>4.54</td></tr>
<tr><td>4</td><td>1.53</td><td>2.13</td><td>2.78</td><td>3.75</td></tr>
<tr><td>5</td><td>1.48</td><td>2.01</td><td>2.57</td><td>3.36</td></tr>
</table></td>
<td>20</td>
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<tr>
<td>6</td>
<td>Given two distance measurements that are independent and have standard deviations $\sigma_1 = 0.20$m and $\sigma_2 = 0.15$m, respectively,
a) Calculate the standard deviations of the sum and of the difference of the two measurements.
b) Calculate the correlation between the sum and the difference.</td>
<td>15</td>
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<td>Total Marks:</td>
<td>100</td>
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