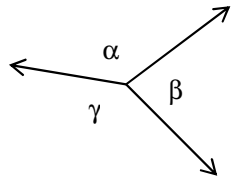
CANADIAN BOARD OF EXAMINERS FOR PROFESSIONAL SURVEYORS
ATLANTIC PROVINCES BOARD OF EXAMINERS FOR LAND SURVEYORS

SCHEDULE I / ITEM 2 **March 2008**

LEAST SQUARES ESTIMATION AND DATA ANALYSIS

Note: This examination consists of 6 questions on 2 pages. **Marks**

Q. No Time: 3 hours Value Earned

Q. No		Value	Earned
1	Define and briefly explain the following terms a) Precision b) Accuracy c) Root mean square error d) Correlation coefficient e) Redundancy of a linear system	10	
2	Given the following mathematical models $f(\lambda, x) = 0 \quad C_\lambda \quad C_x$ where f is the vector of the mathematical model, x is the vector of unknown parameters and C_x is its variance matrix, λ is the vector of observations and C_λ is its variance matrix. a) Provide the linearized form of the given mathematical model. b) Formulate the variation function. c) Derive the least squares solution of the unknown parameters.	15	
3	Given the angle measurements at a station along with their standard deviations: **Angle** | **Measurement** | **Standard Deviation** α | $134^\circ 38' 56''$ | 6.7” β | $83^\circ 17' 35''$ | 9.9” γ | $142^\circ 03' 14''$ | 4.3”  Apply the least squares adjustment to the problem using a) Conditional equations (conditional adjustment) b) Observation equations (parametric adjustment)	30	

4	Given the variance-covariance matrix of the horizontal coordinates (x, y) of a survey station, determine the semi-major, semi-minor axis and the orientation of the standard error ellipse associated with this station. $$C_x = \begin{bmatrix} 0.000532 & 0.000602 \\ 0.000602 & 0.000838 \end{bmatrix} \ m^2$$	10	
5	A baseline of calibrated length (μ) 200.0m is measured 5 times. Each measurement is independent and made with the same precision. The sample mean ($\bar{x}$) and sample standard deviation (s) are calculated from the measurements: $\bar{x}$ = 200.5m s = 0.05m a) Describe the concept and major steps to test the mean value including related mathematical equations and test statistics. b) Test at the 95% level of confidence if the measured distance is significantly different from the calibrated distance. The critical value that might be required in the testing is provided in the following table: **Percentiles of t distribution**	20	
6	Given two distance measurements that are independent and have standard deviations σ_1 = 0.20m and σ_2 = 0.15m, respectively, a) Calculate the standard deviations of the sum and difference of the two measurements. b) Calculate the correlation between the sum and the difference.	15	
	Total Marks:	100	

	t_α			
Degree of freedom	$t_{0.90}$	$t_{0.95}$	$t_{0.975}$	$t_{0.99}$
1	3.08	6.31	12.7	31.8
2	1.89	2.92	4.30	6.96
3	1.64	2.35	3.18	4.54
4	1.53	2.13	2.78	3.75
5	1.48	2.01	2.57	3.36