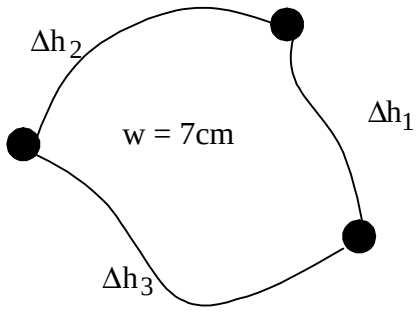
ASSOCIATION OF CANADA LANDS SURVEYORS - BOARD OF EXAMINERS
WESTERN CANADIAN BOARD OF EXAMINERS FOR LAND SURVEYORS
ATLANTIC PROVINCES BOARD OF EXAMINERS FOR LAND SURVEYORS

SCHEDULE I / ITEM 2 **March 2002**

LEAST SQUARES ESTIMATION AND DATA ANALYSIS

Note: This examination consists of __6 _ questions on __2_ pages. **Marks**

Q. No Time: 3 hours Value Earned

Q. No		Value	Earned
1	Define and briefly explain the following terms a) Precision b) Accuracy c) Confidence level d) Redundancy of a linear system e) Unbiasedness of an estimator	10	
2	Given the following mathematical models $f_1(\ell_1, x_1) = 0 \quad C_{\ell_1}$ $f_2(\ell_2, x_1, x_2) = 0 \quad C_{\ell_2} C_{x_2}$ where f_i, x_i, ℓ_i and C_i represent mathematical model vectors, unknown parameter vectors, observation vectors and covariance matrices. a) Formulate the variation function. b) Derive the most expanded form of the least squares normal equation system.	25	
3	Perform a least squares adjustment of the following leveling network in which three height differences Δh_i, i = 1, 2, 3 were observed.  The misclosure w is 7cm. Each Δh_i was measured with a variance of 2.25 cm^2.	20	

<table>
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<td>4</td>
<td>Given the variance-covariance matrix of the horizontal coordinates (x, y) of a survey station, determine the semi-major, semi-minor axis and the orientation of the standard error ellipse associated with this station.
$$C_x = \begin{bmatrix} 0.22 & 0.0246 \\ 0.0246 & 0.14 \end{bmatrix} \ m^2$$</td>
<td>10</td>
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<td>5</td>
<td>A baseline of calibrated length (μ) 100.0m is measured 5 times. Each measurement is independent and made with the same precision. The sample mean ($\bar{x}$) and sample standard deviation (s) are calculated from the measurements:
$\bar{x}$ =100.5m s = 0.05m
a) Describe the major steps to test the mean value.
b) Test at the 5% level of confidence if the measured distance is significantly different from the calibrated distance.
The critical value that might be required in the testing is provided in the following table:
Percentiles of t distribution
<table>
<tr><td></td><td colspan="4">t_α</td></tr>
<tr><td>Degree of freedom</td><td>$t_{0.90}$</td><td>$t_{0.95}$</td><td>$t_{0.975}$</td><td>$t_{0.99}$</td></tr>
<tr><td>1</td><td>3.08</td><td>6.31</td><td>12.7</td><td>31.8</td></tr>
<tr><td>2</td><td>1.89</td><td>2.92</td><td>4.30</td><td>6.96</td></tr>
<tr><td>3</td><td>1.64</td><td>2.35</td><td>3.18</td><td>4.54</td></tr>
<tr><td>4</td><td>1.53</td><td>2.13</td><td>2.78</td><td>3.75</td></tr>
<tr><td>5</td><td>1.48</td><td>2.01</td><td>2.57</td><td>3.36</td></tr>
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<td>20</td>
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<tr>
<td>6</td>
<td>Given the following direct model for y_1 and y_2 as a function of x_1, x_2 and x_3:
$$y_1 = 2x_1 - x_2 - x_3 + 20$$
$$y_2 = x_1 + x_2 - 3x_3 - 50$$
where $x_1 = x_2 = x_3 = 1$ and the covariance matrix of the x's:
$$C_x = \begin{bmatrix} 4 & -2 & -1 \\ -2 & 2 & 1 \\ -1 & 1 & 2 \end{bmatrix}$$
Compute the covariance matrix $\mathbf{C}_y$ for y's.</td>
<td>15</td>
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<td>Total Marks:</td>
<td>100</td>
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