SCHEDULE I / ITEM 1   **October 2007**
MATHEMATICS

Note: This examination consists of 10 questions on 1 page.   **Marks**

Q. No	Time: 3 hours	Value	Earned
1. a)	In Cartesian x, y and z coordinates, what is the equation of a sphere of radius r with centre at (1, 2, 3)?	5	
b)	For some arbitrary point (x_1, y_1, z_1) on the previous sphere, what is the equation of the tangent plane? Briefly explain the steps in deriving the tangent plane equation.	5	
2. a)	In two-dimensional Cartesian coordinates, what is the general equation for a parabola? Explain the meanings of the parameters used.	5	
b)	In two-dimensional Cartesian coordinates, what is the general equation for a hyperbola? Explain the meanings of the parameters used.	5	
3. a)	Given a function $f(x, y, z) = e^{xyz}$, what is the total derivative df(x,y, z)?	5	
b)	What are the partial derivatives $\partial f/\partial x$, $\partial f/\partial y$ and $\partial f/\partial z$ of $f(x, y, z) = e^{xyz}$?	5	
4. a)	For an arbitrary square matrix A, express A as a sum of a symmetric matrix B and a skew-symmetric matrix C. Justify the choice of the matrices B and C.	5	
b)	Give an example of a small matrix A of order 4 with the computations for B and C.	5	
5. a)	What is de Moivre's formula for complex exponentials?	5	
b)	In the complex plane, what are the real and imaginary parts of a complex function sin z?	5	
6. a)	Expand $(1 + 2x)^{1/4}$ into a power series in terms of x. Evaluate the first 3 terms only.	5	
b)	What can be said about the convergence of this series for $(1 + 2x)^{1/4}$?	5	
7. a)	Given three equations x + 2y + 3z = 15, x - 4y + 5z = 18, x + 6y - 7z = 21, what are x, y and z by Gaussian elimination?	5	
b)	What are the x, y and z by applying Cramer's rule to the preceding equations?	5	
8. a)	For an homogeneous algebraic system Ax = 0 for an arbitrary square matrix A, when does it have only the solution x = 0?	5	
b)	For the same homogeneous algebraic system Ax = 0 for an arbitrary square matrix A, when does it have at least one solution x ≠ 0?	5	
9. a)	For some function f(x), $d^2f(x)/dx^2 = \log x$, then what is the general solution f(x)?	5	
b)	For the preceding differential equation and initial conditions f(1) = 1 and df(1)/dx = 0, what is the solution f(x)?	5	
10. a)	For a function g(x,y), given $\partial^2 g(x,y)/\partial x^2 = \cos xy$, what is the general solution g(x,y)?	5	
b)	For a function g(x,y), given $\partial^2 g(x,y)/\partial y^2 = \cos xy$, what is the general solution g(x,y)?	5	
	Total Marks:	100	