ASSOCIATION OF CANADA LANDS SURVEYORS - BOARD OF EXAMINERS WESTERN CANADIAN BOARD OF EXAMINERS FOR LAND SURVEYORS ATLANTIC PROVINCES BOARD OF EXAMINERS FOR LAND SURVEYORS

SCHEDULE I / ITEM 2

October 2005

Marks

LEAST SQUARES ESTIMATION AND DATA ANALYSIS

Note:	This examination consists of 7 questions on 3 pages.
0 N	TT: 2.1

Value Earned <u>Q. No</u> Time: 3 hours Given a leveling network below where A and B are two control points with known height, h₁ and h₂ are two height difference measurements with standard deviation of σ_1 and σ_2 , respectively and $\sigma_1 = 2 \sigma_2$. Determine the value of σ_1 and σ_2 so that the standard deviation of the height solution for point P using least squares adjustment is equal to 2mm. 1 10 h_2 Α B Given the variance-covariance matrix of the measurement vector $\ell = \begin{bmatrix} \ell_1 \\ \ell_2 \end{bmatrix}$: $\mathbf{C}_{\ell} = \begin{vmatrix} \frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} \end{vmatrix}$ 2 10 and two functions of ℓ : x = $\ell_1 + \ell_2$ and y = $3\ell_1$, determine C_{xy}, C_{x\ell}, C_{y\ell} Given the variance-covariance matrix of the horizontal coordinates (x, y) of a survey station, determine the semi-major and semi-minor axes and the orientation of the standard error ellipse associated with this station. 3 $C_{x} = \sigma_{0}^{2} \begin{bmatrix} 0.380 & 0.025 \\ 0.025 & 0.510 \end{bmatrix}$ 10

where $\sigma_0 = 2$ cm.

	Ang	le Meas	urement	Standard Devi	ation	
	α	134	°38'56"	6.7"		
	β	830	² 17'35"	9.9"		
	γ	142	3.14"	4.3		
]	Perform least so	← quares adjustme equations (conc	$\frac{\alpha}{\gamma} \frac{\beta}{\beta}$ ent to the pro-	oblem using		30
	 A baseline of calibrated length (μ) 1153.00m is measured 5 times. Each measurement is independent and made with the same precision. The sample mean (x̄) and sample standard deviation (s) are calculated from the measurements: x̄ =1153.39m s = 0.06m a) Describe the major steps to test the mean value. b) Test at the 10% level of confidence if the measured distance is significantly different from the calibrated distance. The critical value that might be required in the testing is provided in the following table: 					
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6	 Define and explain briefly the following terms: 1) Expectation 2) Unbiasedness of an estimator 3) RMS 4) Null hypothesis and alternative hypothesis 5) Type I error and Type II error 	10	
7	Given the following mathematical models $f_1(\ell_1, x_1) = 0$ C_{ℓ_1} $f_2(\ell_2, x_1, x_2) = 0$ $C_{\ell_2}C_{x_2}$ where f_i , x_i , ℓ_i and C_i represent mathematical model vectors, unknown parameter vectors, observation vectors and covariance matrices. a) Linearize the mathematical models b) Formulate the variation function.	10	
	Total Marks:	100	