CANADIAN BOARD OF EXAMINERS FOR PROFESSIONAL SURVEYORS

C-4 COORDINATE SYSTEMS & MAP PROJECTIONS

March 2014

Although programmable calculators may be used, candidates must show all formulae used, the substitution of values into them, and any intermediate values to 2 more significant figures than warranted by the answer. Otherwise, full marks may not be awarded even though the answer is numerically correct.

Note: This examination consists of 7 questions on 3 pages. **Marks**

Q. No	Time: 3 hours	Value	Earned
1.	The map coordinates of points R and S in a stereographic double projection and the UTM projection are as follows: (see table below)		
	a) Calculate the plane bearings of line RS in the stereographic double projection and UTM projection. Explain two important reasons why the two calculated plane bearings are different.	4	
	b) Calculate (independently) the geodetic bearings of line RS from the plane bearings computed for the stereographic double projection and the UTM projection (assuming the mean radius of the earth in the mapping region, R_m = 6,382,129.599 m). Explain one important reason why the two calculated geodetic bearings are different.	18	
2.	The map projection equations relating map projection coordinates (x, y) with the corresponding geographic coordinates (ϕ, λ) can be given as $x = R\lambda$ $y = R\sin\phi$ where R is the mean radius of the spherical earth. Answer the following questions:		
	a) Calculate the area distortion factor and indicate if this projection is equal-area.	7	
	b) Derive a mathematical expression for the maximum angular distortion at any point in the map projection, and calculate the latitude where the maximum angular distortion reaches 40° on the projection.	6	
3.	According to Torge in his *Geodesy*, the Celestial Reference System (CRS) is an approximation to an inertial system.		
	a) What is an inertial system? Explain the need for it in Geomatics.	4	
	b) Describe the three-dimensional Cartesian coordinate system and the spherical coordinate system, for CRS.	6	

Table for Question 1:

	Stereographic double Projection	**UTM projection**
Point R:	Northing = 7,528,656.555 m, Easting = 2,497,692.180 m Latitude: 46° 45′ 28.0998″ N, Longitude: 66° 31′ 48.7559″ W Grid Convergence = -0° 01′ 19.06″ (T-t) Correction = 0.04″	Northing = 5,181,212.674 m, Easting = 688,610.493 m Latitude: 46° 45′ 28.0998″ N, Longitude: 66° 31′ 48.7559″ W
Point S:	Northing = 7,519,393.255 m, Easting = 2,498,724.831 m	Northing = 5,171,985.701 m, Easting = 689,937.055 m

4.	Explain each of the following (giving clear explanation of each of the terms involved and qualify your answer) : a) One main difference between datum transformation and coordinate transformation. b) One main difference between reference system and reference frame. c) One main difference between horizontal geodetic datum and coordinate reference system. d) One important difference between the CGVD28 and CGVD2013.	3 3 2 3	
5.	Answer all of the following: a) Maps that cover large areas of the globe usually have stated scale called the nominal scale. However, on such maps one simply cannot measure the distance from any point on a map to another, multiply by the nominal scale, and expect to have obtained a correct distance. Explain why. Clearly explain how you would obtain a correct distance from such maps. b) Give two important reasons why the conformality property of UTM projection is attractive to surveyors. c) Discuss the differences and the relationship between a map projection and its map grid.	5 4 5	
6.	a) Explain two fundamental roles of time in positioning applications. b) Explain the differences between the Local Astronomic coordinate system and the Horizon coordinate system with regard to their origins, fundamental planes, and how an object can be positioned in each. c) Explain what natural coordinates are and how they can be determined for a point in space.	4 6 4	
7.	Using well labeled sketches only, illustrate the Mercator and the polar Stereographic projections in the Northern hemisphere; give one sketch for the Mercator projection and the other sketch for the polar Stereographic projection. The sketches must show the representations (in dotted lines) of **loxodrome with bearing 90˚**, and the projections (in bold lines) of the **Equator**, **Central Meridian**, **parallels** and **meridians** with the appropriate relationship between the lines of the graticule clearly illustrated.	16	
		100	

Some potentially useful formulae are given as follows:

$$T - t = \frac{(y_2 - y_1)(x_2 + 2x_1)}{6R_m^2}$$

where $y_i = y_i^{UTM}$; $x_i = x_i^{UTM} - 500{,}000$; R_m is the Gaussian mean radius of the earth; and x_i^{UTM} and y_i^{UTM} are the UTM Easting and Northing coordinates respectively, for point i.

UTM average line scale factor, $\bar{k}_{UTM} = 0.9996\left[1 + \frac{x_u^2}{6R_m^2}\left(1 + \frac{x_u^2}{36R_m^2}\right)\right]$;

where $x_i = x_i^{UTM} - 500{,}000$; $x_u^2 = x_1^2 + x_1 x_2 + x_2^2$

Grid convergence, $\gamma_B = \Delta\lambda \sin\phi \left[1 + \frac{\Delta\lambda^2 \cos^2\phi}{3(20265)^2}\right]$; where $\Delta\lambda = (\lambda - \lambda_0)$ (in arc-seconds) for any given longitude λ with central longitude at λ_0.

$$\alpha = t + \gamma + (T - t)$$

$$X_f = X_G \times K_f + X_0; \qquad Y_f = Y_G \times K_f$$

$$Sf = \frac{R_m}{R_m + H_m}$$

$$x = (N + h)\cos\phi\cos\lambda; \qquad y = (N + h)\cos\phi\sin\lambda$$

$$z = \left[(1 - e^2)N + h\right]\sin\phi; \qquad N = \frac{a}{\sqrt{(1 - e^2\sin^2\phi)}}$$

$$\Delta r^{LG} = R_3(\eta_0 \tan\phi_0)R_2(-\xi_0)R_1(\eta_0) \times \Delta r^{LA}$$

$$\Delta r^G = R_3(\pi - \lambda_0)R_2(\frac{\pi}{2} - \phi_0)P_2 \times \Delta r^{LG}$$

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_B^G = r^G + \Delta r^G$$

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} x_0 \\ y_0 \\ z_0 \end{bmatrix} + (1 + \delta k)\begin{bmatrix} 1 & \gamma & -\beta \\ -\gamma & 1 & \alpha \\ \beta & -\alpha & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\alpha_{ij} = A_{ij} - \eta_i \tan\phi_i - (\xi_i \sin\alpha_{ij} - \eta_i \cos\alpha_{ij})\cot Z_{ij}$$

<u>Distortion Formulas:</u>

Given: $X = f(\phi, \lambda) \qquad Y = g(\phi, \lambda)$

$$m_1^2 = \frac{f_\phi^2 + g_\phi^2}{R^2}; \qquad m_2^2 = \frac{f_\lambda^2 + g_\lambda^2}{R^2\cos^2\phi}; \qquad p = \frac{2(f_\phi f_\lambda + g_\phi g_\lambda)}{R^2\cos\phi}$$

$$\frac{d\Sigma'}{d\Sigma} = m_1 \times m_2 \sin A'_p$$

$$\sin A'_p = \frac{f_\lambda g_\phi - f_\phi g_\lambda}{\sqrt{(f_\lambda g_\phi - f_\phi g_\lambda)^2 + (f_\phi f_\lambda + g_\phi g_\lambda)^2}} \quad ; \qquad \tan\mu_m = \frac{f_\varphi}{g_\varphi}$$

$$\tan\mu_s = \frac{g_\phi \cos\phi\cos A + g_\lambda \sin A}{f_\phi \cos\phi\cos A + f_\lambda \sin A} \quad ; \qquad \tan(180^\circ - A') = \frac{\tan\mu_m - \tan\mu_s}{1 + \tan\mu_m \tan\mu_s}$$

$$\sin\left(\frac{\omega}{2}\right) = \frac{(a - b)}{(a + b)}$$