CANADIAN BOARD OF EXAMINERS FOR PROFESSIONAL SURVEYORS

C-4 COORDINATE SYSTEMS & MAP PROJECTIONS

March 2013

Although programmable calculators may be used, candidates must show all formulae used, the substitution of values into them, and any intermediate values to 2 more significant figures than warranted by the answer. Otherwise, full marks may not be awarded even though the answer is numerically correct.

Note: This examination consists of 6 questions on 3 pages. **Marks**

Q. No	Time: 3 hours	Value	Earned
1.	Explain each of the following (with a demonstration of your understanding of each of the terms involved): a) Differences between the inverse problem of geodetic coordinate transformations and the inverse problem of mapping. b) Main differences between a horizontal datum and a coordinate reference system. c) Three main differences between the geoid and the CGVD28 (including their realization). [Do not be tempted to state, for example, that one is … and the other is not; give complete explanation of what both are as one of the main differences.]	4 3 6	
2.	a) A 3TM zone (with false Easting of 304,800 m and the scale factor of central meridian of 0.99990) and a UTM zone have the same central meridian. Given the 3TM map coordinates of point Q as X = 274,800.000 m, Y = 5,500,000.000 m, calculate the UTM coordinates of the point. b) Explain if the convergence of meridian at point Q in (a) above will be the same in the 3TM projection and the UTM Zone.	4 2	
3.	Given the Universal Transverse Mercator (UTM) map coordinates (in UTM Zone 10) of points B and C as follows: **Station** / **Easting (m)** / **Northing (m)**: B / 564,702.284 / 5,588,965.983 C / 563,836.008 / 5,580,487.376 The other parameters of the map projection are as follows: • Geodetic coordinates of point B are Latitude = 50° 26′ 56.8740″, Longitude = -122° 05′ 19.1800″; • Mean radius of the earth for the region, R_m = 6,382,129.599 m; • Ellipsoidal parameters, a = 6378137.0000 m; e^2 = 0.006694380025; and e'^2 = 0.006739496780. Answer the following: a) Calculate the geodetic azimuth of line BC (to the tenth of a second), and explain the steps and the quantities needed to transform this azimuth into an astronomic azimuth. b) Calculate the ellipsoidal distance between points B and C, and explain the steps and the quantities needed to transform this distance into a mark to mark distance on the ground surface.	16 8	

4.	Answer the following with respect to the (x, y, z) astronomic coordinate system. a) Define the (x, y, z) astronomic coordinate system with regard to its origin and the orientation of its axes. b) Determine (with reasons for each case) which of the following would be affected if the (x, y, z) axes are rotated about the z-axis: astronomic latitude, astronomic longitude, astronomic azimuth. c) Assume that the (x, y, z) axes of an ellipsoid coordinate system are parallel to the (x, y, z) astronomic coordinate system. Explain if the geodetic and astronomic meridian planes for a given point on the Earth's surface are also parallel. Your explanation must demonstrate that you understand what the two meridian planes are.	4 3 4	
5.	a) Discuss the properties of the Polar Stereographic projection and the Mercator projection, with regard to aspects, distortion characteristics and representation of graticule. b) Two of the important considerations in the choice of a suitable map projection for navigation purpose are how the projection represents bearings in relation to navigation compass bearings and how it distorts scale. With regard to these two considerations, discuss the suitability of the Mercator projection and the Polar Stereographic projection for navigational purposes in the Canadian Arctic region (North of the 68^{th} parallel).	15 6	
6.	a) Discuss the terrestrial geocentric (Cartesian) coordinate system and the Local Geodetic coordinate system with regard to the following: i) Description of how a point on the earth surface can be located in each of the systems (or alternatively describe the coordinate systems). ii) Two important disadvantages of each system (considering how one usually interprets their coordinate values in space). b) Given the coordinates of a point 1 in the terrestrial geocentric (Cartesian) system as (X_1, Y_1, Z_1), and the total station measurements from a setup terrain point 1 to 2 as slope distance s_{12}, astronomic azimuth A_{12}, and the zenith angle Z_{12}, describe step by step how you would determine the coordinates (X_2, Y_2, Z_2) of point 2 in the terrestrial geocentric (Cartesian) system, stating what quantities would be needed in each step in order to finally determine the coordinates. c) The GPS-determined position (X, Y, Z) of the corner of a certain parcel of land is to be integrated with the rest of the parcel's boundary that is defined by points whose (Latitude, Longitude, ellipsoidal height) positions are known on the NAD83 (original), and have been derived from a terrestrial network. Explain how this integration can be mathematically done (clearly stating the parameters that may be needed in the process).	6 4 10 5	
		100	

Some potentially useful formulae are given as follows:

$$T - t = \frac{(y_2 - y_1)(x_2 + 2x_1)}{6R_m^2}$$

where $y_i = y_i^{UTM}$; $x_i = x_i^{UTM} - 500{,}000$; R_m is the Gaussian mean radius of the earth; and x_i^{UTM} and y_i^{UTM} are the UTM Easting and Northing coordinates respectively, for point i.

UTM average line scale factor, $\bar{k}_{UTM} = 0.9996\left[1 + \frac{x_u^2}{6R_m^2}\left(1 + \frac{x_u^2}{36R_m^2}\right)\right]$;

where $x_i = x_i^{UTM} - 500{,}000$; $x_u^2 = x_1^2 + x_1 x_2 + x_2^2$

Grid convergence, $\gamma_B = \Delta\lambda \sin\phi\left[1 + \frac{\Delta\lambda^2 \cos^2\phi}{3(20265)^2}\right]$; where $\Delta\lambda = (\lambda - \lambda_0)$ (in arc-seconds) for any given longitude λ with central longitude at λ_0.

$$Sf = \frac{R_m}{R_m + H_m}$$

$$x = (N + h)\cos\phi\cos\lambda$$

$$y = (N + h)\cos\phi\sin\lambda$$

$$z = \left[\left(1 - e^2\right)N + h\right]\sin\phi$$

$$N = \frac{a}{\sqrt{\left(1 - e^2 \sin^2\phi\right)}}$$

$$X_f = X_G \times K_f + X_0$$

$$Y_f = Y_G \times K_f$$

$$\Delta r^{LG} = R_3(\eta_0 \tan\phi_0)R_2(-\xi_0)R_1(\eta_0) \times \Delta r^{LA}$$

$$\Delta r^{G} = R_3(\pi - \lambda_0)R_2\left(\frac{\pi}{2} - \phi_0\right)P_2 \times \Delta r^{LG}$$

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_B^G = r^G + \Delta r^G$$

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} x_0 \\ y_0 \\ z_0 \end{bmatrix} + (1 + \delta k)\begin{bmatrix} 1 & \gamma & -\beta \\ -\gamma & 1 & \alpha \\ \beta & -\alpha & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\alpha_{ij} = A_{ij} - \eta_i \tan\phi_i - \left(\xi_i \sin\alpha_{ij} - \eta_i \cos\alpha_{ij}\right)\cot Z_{ij}$$