CANADIAN BOARD OF EXAMINERS FOR PROFESSIONAL SURVEYORS

C-3 ADVANCED SURVEYING

March 2014

Note: This examination consists of 7 questions and formulae on 7 pages.

Although programmable calculators may be used, candidates must show all formulae used, the substitution of values into them, and any intermediate values to 2 more significant figures than warranted for the answer. Otherwise, full marks may not be awarded even though the answer is numerically correct.

Time: 3 hours

Q. No		Marks Value	Marks Earned
1.	Elementary surveying textbooks suggest that the EODMI additive constant, z_0 in the sense that $s = s' + z_0$, may be uniquely determined using observations [s'_{AB}, s'_{BC}, s'_{AC}] among three collinear points [A, B, and C]. a) Explain how z_0 is calculated using the three s'_{ij}. b) If each s'_{ij} has a standard deviation of $\sigma_{s'}$, derive an expression for the standard deviation of z_0 and explain why this would not be suitable when expecting a value for σ_s which would be compatible with $\sigma_{s'}$.	1 1	
	The standard, ISO 17123-4, recommends using a linear array of 7 points to determine z_0. The array would be set out following the Heerbrugg design for point spacing. c) Explain why this would be preferred rather than using just 3 points, why the Heerbrugg design would be followed rather than a random or uniform spacing, what are the observables, and what are the quantities to be estimated. d) Explain the observation equation relating an s'_{ij} to the appropriate unknowns and what would be done to the output of the EODMI to create the s'_{ij}. e) Explain the statistical tests [at least three] that would be applied *a posteriori* [for each: null hypothesis, alternate hypothesis, statistic, and test].	4 2 3	
	Sometimes standards of government authorities require determining z_0 from measurements on a calibration baseline. f) Explain why, the quantities observed, the quantities estimated and a typical observation equation. g) Explain how the estimation would be different from the 7 point array and what would be the statistical tests involved.	2 2	
2.	A local plane coordinate system was established at the collar of a shaft at a latitude of 60°59'N. At a depth of 2.2 km, an adit runs approximately in a westerly direction. A flat traverse follows the adit with stations along one side. Gyro-azimuths, following the transit method, have been observed at regular intervals in order to "control" the orientation of the adit. The transit method results in an angle, A_g, describing the direction of the gyro zero with respect to North. The equipment and procedures suggest that $\sigma_{Ag} = \pm 3''$. Explain the corrections, with suggestions of their values [especially signs], which should be applied to an A_g, observed at 4 km [westerly] from the shaft in order to convert it to a grid azimuth in the surface coordinate system. If you are not able to calculate a value for a correction, explain what other information would be needed to do so and how it would be obtained.	15	

3.	A campaign of observations, at t_1, can be adjusted to estimate the coordinates of the points involved, based on $\mathbf{l}_1 + \mathbf{v}_1 = \mathbf{A}_1\mathbf{x}_1$ ["**l**" etc. (bold lower case) denote vectors, "**A**" (bold) denotes a matrix]. During a later campaign, at t_2, the observations can be repeated so that $\mathbf{l}_2 + \mathbf{v}_2 = \mathbf{A}_2\mathbf{x}_2$. If there are object points on a sensitive structure, its behaviour can be described geometrically with respect to the reference points, using the displacement field resulting from $\mathbf{d}_x = \mathbf{x}_2 - \mathbf{x}_1$, in which some of the elements of the $\mathbf{d}_x$ vector are for the object points, say $\mathbf{d}_{x\ obj}$. Under certain circumstances, it may be possible to difference the observations, $\mathbf{d}_l = \mathbf{l}_2 - \mathbf{l}_1$, so that the displacement field can be estimated, based on $\mathbf{d}_l + \mathbf{v}_d = \mathbf{A}\mathbf{d}_x$. a) Explain the conditions under which the $\mathbf{d}_x$ can be calculated in the coordinate differencing approach, with respect to the $\mathbf{l}_i$, $\mathbf{A}_i$ and $\mathbf{x}_i$, and the advantages and disadvantages of this approach. b) Explain the conditions under which the observation differencing approach may be followed and its advantages and disadvantages. c) Explain which approach can accommodate geotechnical data and give an example of an appropriate geotechnical observable, r_j, with its observation equation [relating observable to estimables] and an explanation of how the value recorded in the field, r'_i at t_i, becomes the "observation" value, r_j. d) If the monitoring were to endure over a long period of time, say several decades, explain what concerns would arise in each of the two approaches and how best to deal with those concerns.	3 3 6 3	
4.	A flat hanging traverse is to be measured with uniform sight lengths of $100\ m \pm 2\ mm$. There are two "fixed" stations, "A" and "B", plus seven traverse stations, "P1" to "P7" so that "B" and "P1" to "P6" would be occupied while "A" and "P7" would be sighted. One approach is to measure the included horizontal angles [values near $180°$] with $\sigma_\beta = \pm 5''$. An alternative method is to occupy certain stations and to observe the azimuth to the next station using a gyro attachment so that $\sigma_A = \pm 15''$. a) If only included angles were observed, explain the dominant influence on the random positional uncertainty at the end point of the traverse, "P7", and suggest a value and orientation of the uncertainty. b) If azimuths rather than included angles were observed, explain the dominant influence on the random positional uncertainty at the end point of the traverse, "P7", and suggest a value and orientation of the uncertainty. c) If the traverse were along a tunnel, close to one wall, explain the dominant systematic influence and whether included angles or azimuths should be observed.	5 3 2	
5.	As part of a special traverse of "n" angles around a city block, a total station is to be set up along one side of the block, at one station with sight distances of 50 m and 250 m with the angle being very close to 180°. The 50 m sight is nearly horizontal but the 250 m sight is at a slope of 15%. These are the extreme values for this situation. a) Accounting for the effects of centering, leveling, pointing, and reading, recommend an instrument that would be capable of meeting the requirement that the block angular misclosure is not to exceed $n^{1/2}10''$. "Not to exceed" is to be regarded as being at 99%. The values taken in the calculation of the misclosure would the averages from at least two sets [a set being the average of face left and face right sightings]. b) Explain which [centering, leveling, pointing, or reading] is the dominant influence in this situation and why.	10 5	

6.	On the shelf in the company's survey stores, you have found a total station that has not been used for at least 20 years. The manufacturer's claim, following DIN 18723 [or ISO 17123, now], is an angular "accuracy", horizontally or vertically, of $\pm 2''$ and a distance "accuracy" of $\pm$ 2 mm $\pm$ 2 ppm. Since there is no record of any testing or calibration of this particular instrument, explain the steps that you would recommend following to determine whether it is capable of behaving as the manufacturer claimed.	15	
7.	Surveys and Mapping Branch [1978], in its *Specifications and Recommendations for Control Surveys and Survey Markers*, specifies that Special Order Levelling should use an optical mechanical instrument [sensitivity of 10″/2mm, magnification of at least 40x] with a parallel plate micrometer along with double scaled invar staves [usually with 10 mm graduations]. Lengths of sight are not to exceed 50 m and are to be balanced within 5 m. Nonetheless, for Special Order, the discrepancy between independent forward and backward levellings is not to exceed [i.e., $1-\alpha = 99\%$] $\pm 3\text{mm}K^{1/2}$ with K in kilometres. First Order levelling is similar but with 32x, 60 m, 10 m, and $\pm 4\text{mm}K^{1/2}$. Several instruments with appropriate staves are available: i. Wild N3 tilting level [42x, 10″/div (setting accuracy ±0.25″), σ_{ISO}: ±0.2 mm]; ii. Kern GK23 tilting level with micrometer [30x, 20″/div, σ_{ISO}: ±0.5 mm]; iii. Kern GK2A automatic level with micrometer [33x, σ_{ISO}: ±0.3 mm]; iv. Leica NA2 automatic level with micrometer [32x, σ_{ISO}: ±0.3 mm]; and v. Leica DNA03 digital automatic level [24x, setting accuracy ±0.3″, σ_{ISO}: ±0.3 mm] with bar coded invar staves. The "σ_{ISO}" is the standard deviation per 1 km of double run levelling. a) Consider each type of level and, with some numerical substantiation, explain whether it is suitable for Special Order levelling and, if not, whether for First Order. b) Explain how you would ensure that the lengths of sight are balanced in Special Order levelling [50 m ± 5 m] along a route with many successive setups.	10 5	
	Total Marks:	100	

Percentiles of the χ^2 distribution:

	0.90	0.95	0.975	0.99	0.995
1	2.71	3.84	5.02	6.63	7.88
2	4.61	5.99	7.38	9.21	10.60
3	6.25	7.81	9.35	11.34	12.84

Percentiles of the t distribution:

	0.90	0.95	0.975	0.99	0.995
19	1.328	1.729	2.093	2.539	2.861
20	1.325	1.725	2.086	2.528	2.845
21	1.323	1.721	2.080	2.518	2.831
27	1.314	1.703	2.052	2.473	2.771
28	1.313	1.701	2.048	2.467	2.763
29	1.311	1.699	2.045	2.462	2.756

Some potentially useful formulae are given below.

$$\sqrt{\sigma_c^2} \approx \pm 0.001h; \sqrt{\sigma_c^2} = \pm 0.0005h; \sqrt{\sigma_c^2} \leq \pm 0.0005h; \sqrt{\sigma_c^2} \leq \pm 0.0001$$

$$\sigma_{\delta_c}^2 = \frac{\sigma_{c_F}^2 + \sigma_{c_T}^2}{s_{FT}^2}$$

$$\sigma_{\beta_C}^2 = \frac{\sigma_{c_F}^2}{s_F^2} + \frac{\sigma_{c_T}^2}{s_T^2} + \left[\frac{1}{s_F^2} + \frac{1}{s_T^2} - \frac{2}{s_F s_T}\cos\beta\right]\sigma_{c_A}^2$$

$$\sigma_l = \pm 0.2 div;\ \ \sigma_l = \pm 0.02 div;\ \ \sigma_l \le \pm 0.5''$$

$$\sigma_{\beta_l} = \pm\sigma_l\sqrt{\cot^2 z_i + \cot^2 z_j}$$

$$\pm\frac{30''}{M} \le \sigma_p \le \pm\frac{60''}{M};\qquad \sigma_{ps} \approx \frac{70''}{M}$$

$$b = 2a + c;\quad a = \frac{120}{206264.8}\frac{D}{M};\quad 2'' \le c \le 4''$$

$$\sigma_r \ge \pm 0.3 div; \sigma_r = \pm 0.3 div; \sigma_r = \pm 2.5 div; \sigma_r = \pm 0.6''$$

$$\sigma_z^2 = \sigma_{z_l}^2 + \sigma_{z_p}^2 + \sigma_{z_r}^2$$

$$\sigma_{z_l} = \pm\sigma_l$$

$$\sigma_{z_p} = \pm\frac{\sigma_p}{\sqrt{2}}$$

$$\sigma_{z_r} = \pm\frac{\sigma_r}{\sqrt{2}}$$

$$\sin\beta_1 = \frac{b_1\sin\alpha_1}{a};\quad \sin\beta_2 = \frac{b_2\sin\alpha_2}{a}$$

$$\sigma_\beta^2 = \frac{\tan^2\beta}{b^2}\sigma_b^2 + \frac{\tan^2\beta}{a^2}\sigma_a^2 + \left(\frac{b^2}{a^2\cos^2\beta} - \tan^2\beta\right)\sigma_\alpha^2$$

$$\sigma_{y_n}^2 = \sum_{i=1}^{n-1}(x_n - x_i)^2\sigma_{\beta_i}^2;\quad \sigma_{y_n}^2 = \sum_{i=1}^{n-1}(x_{i+1} - x_i)^2\sigma_{\alpha_i}^2$$

$$\sigma_{x_n}^2 = \sigma_s^2\sum_{i=1}^{n}\left(\frac{x_i - x_{i-1}}{s_i}\right)^2$$

$$\sigma_{x_n}^2 = \sum_{i=1}^{n-1}(y_n - y_i)^2\,\sigma_{\beta_i}^2 + \sum_{i=1}^{n-1}\left(\frac{x_{i+1} - x_i}{s_i}\right)^2\sigma_{s_i}^2$$

$$\sigma_{y_n}^2 = \sum_{i=1}^{n-1}(x_n - x_i)^2\,\sigma_{\beta_i}^2 + \sum_{i=1}^{n-1}\left(\frac{y_{i+1} - y_i}{s_i}\right)^2\sigma_{s_i}^2$$

$$\sigma_{x_n y_n} = \sum_{i=1}^{n-1}(y_n - y_i)(x_n - x_i)\sigma_{\beta_i}^2 + \sum_{i=1}^{n-1}\left(\frac{(x_{i+1} - x_i)(y_{i+1} - y_i)}{s_i^2}\right)\sigma_{s_i}^2$$

$\sigma_s^2 = a^2 + b^2 s^2$

$$d\delta = 8" \frac{pS}{T^2}\frac{dT}{dx}$$

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1 atm = 1013.25 mb = 101.325 kPa = 760 torr = 760 mmHg
0 C = 273.15 K
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$$T = \frac{\sum_{i=1}^{n}\left[\left(h_{i+1} - h_i\right)\left(T_i + T_{i+1}\right)\right]}{2\left(h_n - h_1\right)}$$

$$\Delta h_w = \frac{w}{aE}\left(Lh - \frac{h^2}{2}\right)$$

$$n_a = 1 + \frac{0.359474(0.0002945)p}{273.15 + t}$$

$$n_a = 1 + \frac{0.359474(0.0002821)p}{273.15 + t}$$

$$\Delta N_1 = 294.5 - \frac{0.29065p}{1 + 0.00366086t}$$

$$\Delta N_1 = 282.1 - \frac{0.29065p}{1 + 0.00366086t}$$

$$\varepsilon_A = \frac{206264.8}{b}\sqrt{e_1^2 + e_2^2}$$

$$e_i^2 = \left[\frac{e}{2}\right]^2 + [2r]^2 + [0.2mm]^2$$

$$\Delta H = \frac{PH}{aE}; \qquad E = 2.1\text{x}10^6 \text{ kgcm}^{-2}$$

$$T = 2\pi\sqrt{\frac{H}{g}}; \qquad g = 980 \text{ cms}^{-2}$$

$$e = \frac{30hHdv^2}{P}$$

$$r_0 = r_2 - \frac{P_1\left(r_1 - r_2\right)}{P_2 - P_1}$$

$$r = \frac{\pi d^4 E}{64RP}$$

$N = N' + \Delta N$; $\Delta N = ca\Delta t$; $E = A - A_g = t \pm \gamma - A_g$

$$\alpha_1 = \mathrm{A} - \eta \tan\phi$$

$$z = Z + [\xi \cos\alpha_1 + \eta \sin\alpha_1]$$

$$\alpha_2 = \alpha_1 + [\eta \cos\alpha_1 - \xi \sin\alpha_1]\cot z$$

$$\alpha_3 = \alpha_2 + \frac{h}{M_m} e^2 \sin\alpha_2 \cos\alpha_2 \cos^2\phi_{TO}$$

$$\alpha = \alpha_3 - \frac{e^2 s^2 \cos^2\phi_m \sin 2\alpha_3}{12 N_m^2}$$

$$t = \alpha - \gamma - [T - t]$$

$$t = \alpha - \theta - [T - t]$$

$$\theta = \frac{d \tan\phi \left(1 - \varepsilon^2 \sin^2\phi\right)^{\frac{1}{2}}}{a}$$

$$\Delta\gamma = \frac{\Delta E \tan\phi}{R}$$

6378206.4 m, 0.0822718948;   6378137.0 m, 0.081819191

$$\varepsilon = \frac{\Delta\ell}{\ell} = \frac{\Delta s}{s}$$

$$d_x = r_{x_1} - r_{x_2}; \quad d_y = r_{y_1} - r_{y_2}$$

$$\theta_x = \frac{d_x}{s}; \quad \theta_y = \frac{d_y}{s}$$

$$c = [N_0 - N_a]s$$

$$c_{cal} = \frac{s_{std} - s_{obs}}{s_{std}} s_i; \; c_{align} = -\frac{d^2}{2s}; \; c_{temp} = \alpha(t - t_0)s; \; c_{tens} = \frac{P - P_0}{aE} s$$

$$c_{sag} = -\frac{s^3}{24}\left(\frac{mg\cos\theta}{P}\right)^2\left(1 \pm \frac{mgs\sin\theta}{P}\right); \quad c_{sea} = \frac{H}{R + H} s$$

$$\frac{s^2}{\sigma^2} \le \frac{1}{\nu}\chi^2_{\nu,1-\alpha}\,?; \quad \frac{1}{F_{\nu_1,\nu_2,1-\frac{\alpha}{2}}} \le \frac{s_1^2}{s_2^2} \le F_{\nu_1,\nu_2,1-\frac{\alpha}{2}}\,?; \quad \frac{a_\mu}{s_{a_\mu}} \le t_{\nu,1-\frac{\alpha}{2}}\,?$$

$$\left|\frac{\hat{r}_i}{\sigma_{\hat{r}_i}}\right| \le n(0,1), 1 - \frac{\alpha}{2}; \qquad \left|\frac{\hat{r}_i}{\hat{\sigma}_{\hat{r}_i}}\right| \le \tau, \nu, 1 - \frac{\alpha}{2}, \quad \tau_\nu = \frac{\sqrt{\nu}}{\sqrt{\nu - 1 + t_{\nu-1}^2}} t_{\nu-1}$$

$$C_x = \sigma_0^2 \left[C_{x_S}^{-1} + \left(A^T P A \right)_U \right]^{-1}$$

$$\Delta_{f/b} \leq \pm 3mm\sqrt{K};\ \ \Delta_{f/b} \leq \pm 4mm\sqrt{K};\ \ \Delta_{f/b} \leq \pm 8mm\sqrt{K};\ \ \Delta_{f/b} \leq \pm 24mm\sqrt{K}$$

$$\sigma_{r_l} = \pm d\sigma_l; \qquad \sigma_{r_{pr}} = \pm\frac{45"}{M}d, \quad d > 20m; \quad \sigma_{r_{pr}} = \pm\frac{30"}{M}d, \quad d \leq 20m$$

$d_{y1} = r_{1,1} - r_{2,1}$; $d_{y2} = r_{1,2} - r_{2,2}$; $\Delta y = d_{y2} - d_{y1}$

$$T = \frac{\Delta y}{\Delta H}$$

$$s_{ij} + z_0 = x_j - x_i;\ ks_{ij} + z_0 = x_j - x_i;\ s = s' + s'\Delta N$$

$$n_{obs} = \frac{n_{pts}\left(n_{pts} - 1\right)}{2}$$

`c+r = 0.0675 K`^{2}

$$A = iU; \quad B_0 = \frac{1}{15}[C_0 - 6A - U]; \quad D = \frac{U}{36}$$

$1to2 : A + 1B + 3D$

$2to3 : A + 3B + 7D$

$3to4 : A + 5B + 11D$

$4to5 : A + 4B + 9D$

$5to6 : A + 2B + 5D$

$6to7 : A + D$

$$d_4 = 2R\arcsin\sqrt{\frac{R^2\sin^2(d_1\frac{k}{2R}) - k^2\frac{(H_2 - H_1)^2}{4}}{k^2(R + H_1)(R + H_2)}}$$

$$d_4 = R\arctan\left[\frac{d_2\sin(z_1 + \varepsilon_1 + \delta)}{R + H_1 + d_2\cos(z_1 + \varepsilon_1 + \delta)}\right]$$

$$x = \frac{-a}{b} + \frac{\hat{y}}{b}$$

$$s_a = s_0\sqrt{\frac{\sum x^2}{n\sum x^2 - \left(\sum x\right)^2}} \quad and \quad s_b = \frac{s_0}{\sqrt{\frac{\sum x^2 - \left(\sum x\right)^2}{n}}} \quad with$$

$$\hat{\sigma}_0^2 = s_0^2 = \frac{\sum\left(a + bx - y\right)^2}{n - 2}$$

$$\hat{\sigma}_{z_0}^2 = \hat{\sigma}_0^2\frac{6}{\left(N-1\right)\left(N-2\right)} \quad with \quad \hat{\sigma}_0^2 = \frac{\sum v^2}{n - u}$$