CANADIAN BOARD OF EXAMINERS FOR PROFESSIONAL SURVEYORS

C-3 ADVANCED SURVEYING <u>March 2013</u>

Note: This examination consists of 7 questions and formulae on 6 pages.

Although programmable calculators may be used, candidates must show all formulae used, the substitution of values into them, and any intermediate values to 2 more significant figures than warranted for the answer. Otherwise, full marks may not be awarded even though the answer is numerically correct.

Q. No	Time: 3 hours	Marks Value	Marks Earned
1.	In his *Introduction to EDM*, Rüeger offers solutions, using combinations of summations, for the additive constant and scale factor from calibration baseline observations and for the additive constant from a linear array. For a calibration baseline, the solution uses linear regression in the form $\hat{y} = a + bx$.		
	a) Explain which, of the known pillar distances and of the observed distances, is the dependent variable and the independent variable in the regression and why.	5	
	b) Using the estimated values of "a" and "b", explain how the additive constant, z_0, and the scale factor, k, are calculated.	5	
	c) Explain whether these are rigorous least squares estimations and why.	5	
	d) The standard deviations, σ_{z0} and σ_k, can be estimated from the regression. Explain how this is done and why.	5	
	For a linear array, the solution involves calculation of the adjusted distances and then misclosures. This has been simplified for linear arrays with a particular number of points. E.g., with 7 points, the additive constant is $z_0 = -\frac{1}{35}\left[5\left(s_{12}+s_{23}+s_{34}+s_{45}+s_{56}+s_{67}-s_{17}\right)+\right]$ $-\frac{1}{35}\left[3\left(s_{13}+s_{35}+s_{57}-s_{27}+s_{24}+s_{46}-s_{16}\right)\right]$ $-\frac{1}{35}\left[\left(s_{14}+s_{47}-s_{37}+s_{36}-s_{26}+s_{25}-s_{15}\right)\right]$		
	e) Explain why this is a rigorous least squares estimation and what assumptions are the basis for that.	5	
	f) From the 28 distances on an 8 point linear array, the additive constant, z_0, was estimated to be -0.91 mm with the standard deviation of an observation of unit weight being estimated to be ±1.14 mm. Explain whether the value of z_0 is significant at 95%.	5	
2.	A local plane coordinate system was established at the collar of a shaft at a latitude of 62°20'N. At a depth of 2 km, an adit runs approximately in an easterly direction. A flat traverse follows the adit with stations along one side. Gyro-azimuths, following the transit method, have been observed at regular intervals in order to "control" the orientation of the adit. The transit method results in an angle, A_g, describing the direction of the gyro zero with respect to North. The equipment and procedures suggest that $\sigma_{Ag} = \pm 3''$. Explain the corrections, with suggestions of their values [especially signs], which should be applied to an A_g, observed at 4 km [easterly] from the shaft in order to convert it to a grid azimuth in the surface coordinate system. If you are not able to calculate a value for a correction, explain what other information would be needed to do so and how it would be obtained.	15	

3.	A campaign of observations, at t_1, can be adjusted to estimate the coordinates of the points involved, based on $\mathbf{l}_1 + \mathbf{v}_1 = \mathbf{A}_1\mathbf{x}_1$ ["**l**" etc. (bold) denote vectors, "**A**" (bold) denotes a matrix]. During a later campaign, at t_2, the observations can be repeated so that $\mathbf{l}_2 + \mathbf{v}_2 = \mathbf{A}_2\mathbf{x}_2$. If there are object points on a sensitive structure, its behaviour can be described geometrically with respect to the reference points, using the displacement field resulting from $\mathbf{d}_x = \mathbf{x}_2 - \mathbf{x}_1$, in which some of the elements of the $\mathbf{d}_x$ vector are for the object points, say $\mathbf{d}_{x\ obj}$. Under certain circumstances, it may be possible to difference the observations, $\mathbf{d}_l = \mathbf{l}_2 - \mathbf{l}_1$, so that the displacement field can be estimated, based on $\mathbf{d}_l + \mathbf{v}_d = \mathbf{A}\mathbf{d}_x$. a) Explain the conditions under which the $\mathbf{d}_x$ can be calculated in the coordinate differencing approach, with respect to the $\mathbf{l}_i$, $\mathbf{A}_i$ and $\mathbf{x}_i$, and the advantages and disadvantages of this approach. b) Explain the conditions under which the observation differencing approach may be followed and its advantages and disadvantages. c) Explain which approach can accommodate geotechnical data and give an example of an appropriate geotechnical observable, r_j, with its observation equation [relating observable to estimables] and an explanation of how the value recorded in the field, r_i at t_i, becomes the "observation" value, r_j. d) If the monitoring were to endure over a long period of time, say several decades, explain what concerns would arise in each of the two approaches and how best to deal with those concerns.	2 2 4 2	
4.	A flat hanging traverse is to be measured with uniform sight lengths of 100 m ±2 mm. There are two "fixed" stations, "A" and "B", plus seven traverse stations, "P1" to "P7" so that "B" and "P1" to "P6" would be occupied while "A" and "P7" would be sighted. One approach is to measure the included horizontal angles [values near 180°] with $\sigma_\beta = \pm 5''$. An alternative method is to occupy certain stations and to observe the azimuth to the next station using a gyro attachment so that $\sigma_A = \pm 15''$. a) If only included angles were observed, explain the dominant influence on the random positional uncertainty at the end point of the traverse, "P7", and suggest a value and orientation of the uncertainty. b) If azimuths rather than included angles were observed, explain the dominant influence on the random positional uncertainty at the end point of the traverse, "P7", and suggest a value and orientation of the uncertainty.	6 4	
5.	As part of a special traverse of "n" angles around a city block, a total station is to be set up along one side of the block, at one station with sight distances of 50 m and 200 m with the angle being very close to 180°. The 50 m sight is nearly horizontal but the 200 m sight is at a slope of 15%. These are the extreme values for this situation. a) Accounting for the effects of centering, leveling, pointing, and reading, recommend an instrument that would be capable of meeting the requirement that the block angular misclosure is not to exceed $n^{1/2}10''$. "Not to exceed" is to be regarded as being at 99%. The values taken in the calculation of the misclosure would the averages from at least two sets [a set being the average of face left and face right sightings]. b) Explain which [centering, leveling, pointing, or reading] is the dominant influence in this situation and why.	10 5	

6.	On the shelf in the company's survey stores, you have found a total station that has not been used for at least 20 years. The manufacturer's claim, following DIN 18723 [or ISO 17123, now], is an angular "accuracy", horizontally or vertically, of ± 5" and a distance "accuracy" of ± 3 mm ± 3 ppm. Since there is no record of any testing or calibration of this particular instrument, explain the steps that you would recommend following to determine whether it is capable of behaving as the manufacturer claimed.	10	
7.	Deformation monitoring often involves geotechnical, as well as geodetic, observables. A repeated extensometer observation yields a change, $\Delta s = r_j - r_i$, over a period of $\Delta t = t_j - t_i$, between two anchor or reference points that are a distance "s" apart. A typical single extensometer reading, r_i or r_j, is ± 0.05 mm. Determine the standard deviation of "s" that would be compatible with the capability of the extensometer if strain, ε, were to be derived from the repeated observations.	10	
	Total Marks:	100	

Percentiles of the χ^2 distribution:

	0.90	0.95	0.975	0.99	0.995
1	2.71	3.84	5.02	6.63	7.88
2	4.61	5.99	7.38	9.21	10.60
3	6.25	7.81	9.35	11.34	12.84

Percentiles of the t distribution:

	0.90	0.95	0.975	0.99	0.995
19	1.328	1.729	2.093	2.539	2.861
20	1.325	1.725	2.086	2.528	2.845
21	1.323	1.721	2.080	2.518	2.831
27	1.314	1.703	2.052	2.473	2.771
28	1.313	1.701	2.048	2.467	2.763
29	1.311	1.699	2.045	2.462	2.756

Some potentially useful formulae are given below.

$$\sqrt{\sigma_c^2} \approx \pm 0.001h; \sqrt{\sigma_c^2} = \pm 0.0005h; \sqrt{\sigma_c^2} \le \pm 0.0005h; \sqrt{\sigma_c^2} \le \pm 0.0001$$

$$\sigma_{\delta_c}^2 = \frac{\sigma_{c_F}^2 + \sigma_{c_T}^2}{s_{FT}^2}$$

$$\sigma_{\beta_C}^2 = \frac{\sigma_{c_F}^2}{s_F^2} + \frac{\sigma_{c_T}^2}{s_T^2} + \left[\frac{1}{s_F^2} + \frac{1}{s_T^2} - \frac{2}{s_F s_T}\cos\beta\right]\sigma_{c_A}^2$$

$$\sigma_l = \pm 0.2 div; \;\; \sigma_l = \pm 0.02 div; \;\; \sigma_l \le \pm 0.5''$$

$$\sigma_{\beta_l} = \pm \sigma_l \sqrt{\cot^2 z_i + \cot^2 z_j}$$

$$\pm \frac{30''}{M} \le \sigma_p \le \pm \frac{60''}{M}; \qquad \sigma_{ps} \approx \frac{70''}{M}$$

$$b = 2a + c; \quad a = \frac{120}{206264.8}\frac{D}{M}; \quad 2'' \le c \le 4''$$

$$\sigma_r \ge \pm 0.3 div; \sigma_r = \pm 0.3 div; \sigma_r = \pm 2.5 div; \sigma_r = \pm 0.6''$$

$$\sigma_z^2 = \sigma_{z_l}^2 + \sigma_{z_p}^2 + \sigma_{z_r}^2$$

$$\sigma_{z_l} = \pm\sigma_l$$

$$\sigma_{z_p} = \pm\frac{\sigma_p}{\sqrt{2}}$$

$$\sigma_{z_r} = \pm\frac{\sigma_r}{\sqrt{2}}$$

$$\sin\beta_1 = \frac{b_1 \sin\alpha_1}{a}; \quad \sin\beta_2 = \frac{b_2 \sin\alpha_2}{a}$$

$$\sigma_\beta^2 = \frac{\tan^2\beta}{b^2}\sigma_b^2 + \frac{\tan^2\beta}{a^2}\sigma_a^2 + \left(\frac{b^2}{a^2\cos^2\beta} - \tan^2\beta\right)\sigma_\alpha^2$$

$$\sigma_{y_n}^2 = \sum_{i=1}^{n-1}(x_n - x_i)^2\sigma_{\beta_i}^2; \quad \sigma_{y_n}^2 = \sum_{i=1}^{n-1}(x_{i+1} - x_i)^2\sigma_{\alpha_i}^2$$

$\sigma_s^2 = a^2 + b^2 s^2$

$$d\delta = 8'' \frac{pS}{T^2}\frac{dT}{dx}$$

1 atm = 1013.25 mb = 101.325 kPa = 760 torr = 760 mmHg
0 C = 273.15 K

$$T = \frac{\sum_{i=1}^{n}\left[(h_{i+1} - h_i)(T_i + T_{i+1})\right]}{2(h_n - h_1)}$$

$$\Delta h_w = \frac{w}{aE}\left(Lh - \frac{h^2}{2}\right)$$

$$n_a = 1 + \frac{0.359474(0.0002945)p}{273.15 + t}$$

$$n_a = 1 + \frac{0.359474(0.0002821)p}{273.15 + t}$$

$$\Delta N_1 = 294.5 - \frac{0.29065p}{1 + 0.00366086t}$$

$$\Delta N_1 = 282.1 - \frac{0.29065p}{1 + 0.00366086t}$$

$$\varepsilon_A = \frac{206264.8}{b}\sqrt{e_1^2 + e_2^2}$$

$$e_i^2 = \left[\frac{e}{2}\right]^2 + [2r]^2 + [0.2mm]^2$$

$$\Delta H = \frac{PH}{aE}; \qquad E = 2.1\text{x}10^6 \text{ kgcm}^{-2}$$

$$T = 2\pi\sqrt{\frac{H}{g}}; \qquad g = 980 \text{ cms}^{-2}$$

$$e = \frac{30hHdv^2}{P}$$

$$r_0 = r_2 - \frac{P_1(r_1 - r_2)}{P_2 - P_1}$$

$$r = \frac{\pi d^4 E}{64RP}$$

N = N' + ΔN; ΔN = caΔt; E = A - A_g = t ± γ - A_g

$$\theta = \frac{d\tan\phi\left(1-\varepsilon^2\sin^2\phi\right)^{\frac{1}{2}}}{a}$$

$$\Delta\gamma = \frac{\Delta E\tan\phi}{R}$$

6378206.4 m, 0.0822718948; 6378137.0 m, 0.081819191

$$\varepsilon = \frac{\Delta\ell}{\ell} = \frac{\Delta s}{s}$$

$$d_x = r_{x_1} - r_{x_2}; \quad d_y = r_{y_1} - r_{y_2}$$

$$\theta_x = \frac{d_x}{s}; \quad \theta_y = \frac{d_y}{s}$$

$$c = [N_0 - N_a]s$$

$$c_{cal} = \frac{s_{std} - s_{obs}}{s_{std}} s_i; \ c_{align} = -\frac{d^2}{2s}; \ c_{temp} = \alpha(t - t_0)s; \ c_{tens} = \frac{P - P_0}{aE}s$$

$$c_{sag} = -\frac{s^3}{24}\left(\frac{mg\cos\theta}{P}\right)^2\left(1 \pm \frac{mgs\sin\theta}{P}\right); \quad c_{sea} = \frac{H}{R+H}s$$

$$\frac{s^2}{\sigma^2} \le \frac{1}{\nu}\chi^2_{\nu,1-\alpha}?; \quad \frac{1}{F_{\nu_1,\nu_2,1-\frac{\alpha}{2}}} \le \frac{s_1^2}{s_2^2} \le F_{\nu_1,\nu_2,1-\frac{\alpha}{2}}?; \quad \frac{a_\mu}{s_{a_\mu}} \le t_{\nu,1-\frac{\alpha}{2}}?$$

$$C_x = \sigma_0^2\left[C_{x_S}^{-1} + \left(A^T PA\right)_U\right]^{-1}$$

$$\Delta_{f/b} \le \pm 3mm\sqrt{K}; \ \Delta_{f/b} \le \pm 4mm\sqrt{K}; \ \Delta_{f/b} \le \pm 8mm\sqrt{K}; \ \Delta_{f/b} \le \pm 24mm\sqrt{K}$$

$d_{y1} = r_{1,1} - r_{2,1}$; $d_{y2} = r_{1,2} - r_{2,2}$; $\Delta y = d_{y2} - d_{y1}$

$$T = \frac{\Delta y}{\Delta H}$$

$$s_{ij} + z_0 = x_j - x_i; ks_{ij} + z_0 = x_j - x_i; s = s' + s'\Delta N$$

`c+r = 0.0675 K`^{2}

$$A = iU; \quad B_0 = \frac{1}{15}[C_0 - 6A - U]; \quad D = \frac{U}{36}$$

$$1to2 : A + 1B + 3D$$
$$2to3 : A + 3B + 7D$$
$$3to4 : A + 5B + 11D$$
$$4to5 : A + 4B + 9D$$
$$5to6 : A + 2B + 5D$$
$$6to7 : A + D$$

$$d_4 = 2R\arcsin\sqrt{\frac{R^2 \sin^2(d_1 \frac{k}{2R}) - k^2 \frac{(H_2 - H_1)^2}{4}}{k^2(R + H_1)(R + H_2)}}$$

$$d_4 = R\arctan\left[\frac{d_2 \sin(z_1 + \varepsilon_1 + \delta)}{R + H_1 + d_2 \cos(z_1 + \varepsilon_1 + \delta)}\right]$$

$$x = \frac{-a}{b} + \frac{\hat{y}}{b}$$

$$s_a = s_0 \sqrt{\frac{\sum x^2}{n\sum x^2 - \left(\sum x\right)^2}} \quad and$$

$$s_b = \frac{s_0}{\sqrt{\frac{\sum x^2 - \left(\sum x\right)^2}{n}}} \quad with$$

$$\hat{\sigma}_0^2 = s_0^2 = \frac{\sum \left(a + bx - y\right)^2}{n - 2}$$

$$\hat{\sigma}_{z_0}^2 = \hat{\sigma}_0^2 \frac{6}{\left(N - 1\right)\left(N - 2\right)} \quad with$$

$$\hat{\sigma}_0^2 = \frac{\sum v^2}{n - u}$$