ASSOCIATION OF CANADA LANDS SURVEYORS - BOARD OF EXAMINERS
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SCHEDULE I / ITEM 2 **September 2002**

LEAST SQUARES ESTIMATION AND DATA ANALYSIS

Note: This examination consists of __6 _ questions on __2_ pages.

Time: 3 hours

Q. No		Marks Value	Marks Earned
1.	Define and briefly explain the following terms a) Null hypothesis and alternative hypothesis b) Type I error and Type II error c) Variance and mean square error d) Statistical independence and uncorrelation	10	
2.	Given the following mathematical models $f_1(\ell_1, x_1) = 0 \quad C_{\ell_1} \quad C_{x_1}$ $f_2(\ell_2, x_1, x_2) = 0 \quad C_{\ell_2} \quad C_{x_2}$ where f_i, x_i, ℓ_i and C_i represent mathematical model vectors, unknown parameter vectors, observation vectors and covariance matrices. a) Formulate the variation function. b) Derive the most expanded form of the least squares normal equation system.	30	
3.	Given the variance-covariance matrix of the horizontal coordinates (x, y) of a survey station, determine the semi-major, semi-minor axis and the orientation of the standard error ellipse associated with this station. $C_x = \begin{bmatrix} 0.0484 & 0.0246 \\ 0.0246 & 0.0196 \end{bmatrix} m^2$	10	

4. Given in the following is a diagram of a leveling network to be adjusted. **30**

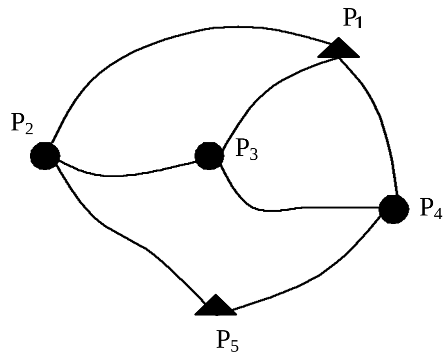

Table 1 contains all of the height difference measurements Δh_{ij} necessary to carry out the adjustment. Assume that all observations have the same standard deviation $s_{\Delta h_{ij}}$ = 0.04m. P1 and P5 are fixed points with known elevations of H_1 = 107.50m and H_5 = 101.00m. Perform least squares adjustment on the leveling network and compute:

a) the adjusted elevations of point 2, 3 and 4;

b) the observation residuals;

c) the adjusted height differences; and

d) the a-posteriori variance-covariance factor $\hat{s}_0$.

Table 1

No.	**i**	**j**	Δh_{ij}
1	P2	P1	1.34
2	P1	P3	-5.00
3	P1	P4	-2.25
4	P2	P3	-3.68
5	P5	P2	5.10
6	P3	P4	2.70
7	P4	P5	-4.13

5. Given the following direct model for y_1 and y_2 as a function of x_1 and x_2:

$$y_1 = \sqrt{x_1^2 - x_2^2}$$

$$y_2 = \sin^{-1}\left(\frac{x_2}{x_1}\right)$$

where $x_1 = 5$ and $x_2 = 3$ and the covariance matrix of the x’s:

$$C_x = \begin{bmatrix} 4 & -2 \\ -2 & 3 \end{bmatrix}$$

Compute the covariance matrix C_y for y’s. **20**

Total Marks: 100