CANADIAN BOARD OF EXAMINERS FOR PROFESSIONAL SURVEYORS

C-2 LEAST SQUARES ESTIMATION & DATA ANALYSIS

October 2012

Although programmable calculators may be used, candidates must show all formulae used, the substitution of values into them, and any intermediate values to 2 more significant figures than warranted for the answer. Otherwise, full marks may not be awarded even though the answer is numerically correct.

Note: This examination consists of 6 questions on 3 pages.

Time: 3 hours

Q. No		Marks Value	 Earned
1.	Briefly explain the following terms (2 marks each) a) Precision b) Accuracy c) Root mean square error d) Correlation coefficient e) Redundancy of a linear system	10	
2.	Given the following mathematical models $f(\ell, x) = 0 \quad C_\ell \quad C_x$ where f is the vector of the mathematical model, x is the vector of unknown parameters and C_x is its variance matrix, ℓ is the vector of observations and C_ℓ is its variance matrix. a) Provide the linearized form of the given mathematical model. b) Formulate the variation function. c) Derive the least squares solution of the unknown parameters.	5 5 10	
3.	Given the variance-covariance matrix of the horizontal coordinates (x, y) of a survey station, determine the semi-major, semi-minor axis and the orientation of the standard error ellipse associated with this station. $C_x = \begin{bmatrix} 0.0484 & 0.0246 \\ 0.0246 & 0.0196 \end{bmatrix} \text{ m}^2$	10	

4.

Given two distance measurements that are independent and have standard deviations $\sigma_1 = 0.20\text{m}$ and $\sigma_2 = 0.15\text{m}$, respectively,

a) Calculate the standard deviations of the sum and difference of the two measurements. 7.5

b) Calculate the correlation between the sum and the difference. 7.5

5.

The following residual vector $\hat{v}$ and estimated covariance matrix $C_{\hat{v}}$ were computed from a least squares adjustment using five independent observations with a standard deviation of $\sigma = 2$ mm and a degree of freedom $\upsilon = 2$:

$$\hat{v} = \begin{bmatrix} 15 & 2 & -3 & 10 \end{bmatrix} \qquad \text{(mm)}$$

$$C_{\hat{v}} = \begin{bmatrix} 15 & 1 & 3 & -2 \\ 1 & 7 & -1 & 3 \\ 3 & -1 & 4 & -1 \\ -2 & 3 & -1 & 2 \end{bmatrix} \qquad (\text{mm}^2)$$

Given $\alpha = 0.02$,

a) Conduct a global test to decide if there exists any outlier or not. 5

b) Conduct local tests to locate possible outlier(s). 10

The critical values that might be required in the testing are provided in the following tables:

α	0.001	0.01	0.02	0.05	0.10
$\chi^2_{\alpha,\ \upsilon=2}$	13.82	9.21	7.82	5.99	4.61

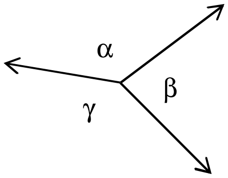
α	0.001	0.002	0.003	0.004	0.005	0.01	0.05
K_α	3.09	2.88	2.75	2.65	2.58	2.33	1.64

where $\chi^2_{\alpha,\ \upsilon=2}$ is determined by the equation $\alpha = \int_{\chi^2_{\alpha,\ \upsilon=2}}^{\infty} \chi^2(x)dx$ and K_α is determined by the equation $\alpha = \int_{K_\alpha}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-x^2/2} dx$.

6.

Given the angle measurements at a station along with their standard deviations:

Angle	**Measurement**	**Standard Deviation**
α	134°38′56″	6.7″
β	83°17′35″	9.9″
γ	142°03′14″	4.3″



Apply the least squares adjustment to the problem using

a) Conditional equations (conditional adjustment). 15

b) Observation equations (parametric adjustment). 15

Total Marks: 100