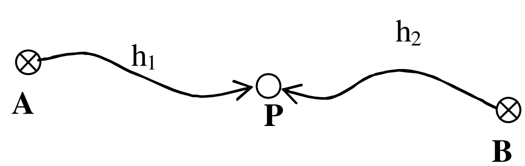
CANADIAN BOARD OF EXAMINERS FOR PROFESSIONAL SURVEYORS

C-2 LEAST SQUARES ESTIMATION & DATA ANALYSIS **October 2010**

Note: This examination consists of 8 questions on 3 pages.

Although programmable calculators may be used, candidates must show all formulae used, the substitution of values into them, and any intermediate values to 2 more significant figures than warranted for the answer. Otherwise, full marks may not be awarded even though the answer is numerically correct. **Marks**

Q. No	Time: 3 hours	Value	Earned
1.	Define and explain briefly the following terms: a) Precision b) Accuracy c) Correlation coefficient d) Redundancy of a linear system e) Type II errors in statistical testing	10	
2.	Given the leveling network below where A and B are known points, h_1 and h_2 are two height difference measurements with standard deviation of σ_1 and σ_2, respectively and $\sigma_1 = 2\,\sigma_2$. Determine the value of σ_1 and σ_2 so that the standard deviation of the height solution at P using least squares adjustment is equal to 2cm. 	10	
3.	Given the following mathematical model $f(\ell, x) = 0 \quad C_\ell \quad C_x$ where f is the vector of mathematical models, x is the vector of unknown parameters and C_x is its variance matrix, ℓ is the vector of observations and C_ℓ is its variance matrix. a) Linearize the mathematical model. b) Formulate the variation function. c) Derive the least squares normal equation.	15	

4.	Given the variance-covariance matrix of the horizontal coordinates (x, y) of a survey station, determine the semi-major, semi-minor axis and the orientation of the standard error ellipse associated with this station. $$C_x = \begin{bmatrix} 0.000532 & 0.000602 \\ 0.000602 & 0.000838 \end{bmatrix} \ m^2$$	10	
5.	Given the variance-covariance matrix of the measurement vector $\ell = \begin{bmatrix} \ell_1 \\ \ell_2 \end{bmatrix}$: $$C_\ell = \begin{bmatrix} \frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} \end{bmatrix}$$ and two functions of $\ell : x = \ell_1 + \ell_2$ and $y = 3\ell_1$, determine $C_{xy}, C_{x\ell}, C_{y\ell}$	10	
6.	Given the sample unit variance obtained from the adjustment of a geodetic network $\hat{\sigma}_0^2 = 0.55\,\text{cm}^2$ with a degree of freedom $\upsilon = 3$ and the a-priori standard deviation $\sigma_0 = 0.44\,\text{cm}$, conduct a statistic test to decide if the adjustment result is acceptable with a significance level of $\alpha = 5\%$. Provide the major test steps and explain the conclusion. The critical values that might be required in the testing are provided in the following table: <table><tr><td>α</td><td>0.001</td><td>0.01</td><td>0.025</td><td>0.05</td><td>0.10</td></tr><tr><td>$\chi^2_{\alpha,\ \upsilon=3}$</td><td>16.26</td><td>11.34</td><td>9.35</td><td>7.82</td><td>6.25</td></tr></table>where $\chi^2_{\alpha,\ \upsilon=3}$ is determined by the equation $\alpha = \int_{\chi^2_{\alpha,\ \upsilon=3}}^{\infty} \chi^2(x)dx$.	10	

7. A baseline of calibrated length (μ) 200.0m is measured 5 times. Each measurement is independent and made with the same precision. The sample mean ($\bar{x}$) and sample standard deviation (s) are calculated from the measurements:

$\bar{x} = 200.5$m $s = 0.05$m

Test at the 95% level of confidence if the measured distance is significantly different from the calibrated distance.

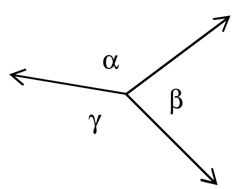
The critical value that might be required in the testing is provided in the following table:

	t_α			
Degree of freedom	$t_{0.90}$	$t_{0.95}$	$t_{0.975}$	$t_{0.99}$
1	3.08	6.31	12.7	31.8
2	1.89	2.92	4.30	6.96
3	1.64	2.35	3.18	4.54
4	1.53	2.13	2.78	3.75
5	1.48	2.01	2.57	3.36

10

8. Given the angle measurements at a station along with their standard deviations:

Angle	**Measurement**	**Standard Deviation**
α	$134^\circ 38' 56''$	$6.7''$
β	$83^\circ 17' 35''$	$9.9''$
γ	$142^\circ 03' 14''$	$4.3''$



Perform least squares adjustment to the problem using

a) Conditional equations (conditional adjustment)
b) Observation equations (parametric adjustment)

25

Total Marks: 100