CANADIAN BOARD OF EXAMINERS FOR PROFESSIONAL SURVEYORS

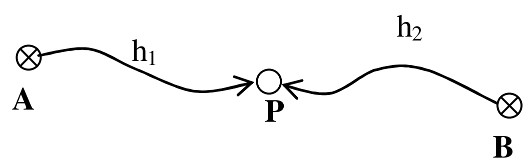
SCHEDULE I / ITEM 2 **October 2009**

LEAST SQUARES ESTIMATION & DATA ANALYSIS

Although programmable calculators may be used, candidates must show all formulae used, the substitution of values into them, and any intermediate values to 2 more significant figures than warranted for the answer. Otherwise, full marks may not be awarded even though the answer is numerically correct.

Note: This examination consists of 8 questions on 3 pages.

Time: 3 hours

Q. No		Marks Value	Marks Earned
1.	Define and explain briefly the following terms: a) Standard deviation b) Variance c) Precision d) Accuracy e) Redundancy of a linear system	10	
2.	Given a leveling network below where A and B are known points, h_1 and h_2 are two height difference measurements with standard deviation of σ_1 and σ_2, respectively and $\sigma_1 = 2\sigma_2$. Determine the value of σ_1 and σ_2 so that the standard deviation of the height solution at P using least squares adjustment is equal to 2cm. 	10	
3.	Given the following mathematical model $f(\ell, x) = 0 \quad C_\ell \quad C_x$ where f is the vector of mathematical models, x is the vector of unknown parameters and C_x is its variance matrix, ℓ is the vector of observations and C_ℓ is its variance matrix. a) Linearize the mathematical model b) Formulate the variation function c) Derive the least squares normal equation d) Derive the least squares solution of the unknown parameters.	15	
4.	Given the variance-covariance matrix of the horizontal coordinates (x, y) of a survey station, determine the semi-major, semi-minor axis and the orientation of the standard error ellipse associated with this station.	10	

$$C_x = \begin{bmatrix} 0.000532 & 0.000602 \\ 0.000602 & 0.000838 \end{bmatrix} \ m^2$$

5.

Given the variance-covariance matrix of the measurement vector $\ell = \begin{bmatrix} \ell_1 \\ \ell_2 \end{bmatrix}$:

$$C_\ell = \begin{bmatrix} \frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} \end{bmatrix}$$

and two functions of ℓ: $x = \ell_1 + \ell_2$ and $y = 3\ell_1$, determine $C_{xy}, C_{x\ell}, C_{y\ell}$

10

6.

Given the sample unit variance obtained from the adjustment of a geodetic network $\hat{\sigma}_0^2 = 0.55\,cm^2$ with a degree of freedom $\upsilon = 3$ and the a-priori standard deviation $\sigma_0 = 0.44\,cm$, conduct a statistic test to decide if the adjustment result is acceptable with a significance level of $\alpha = 5\%$. Provide the major test steps and explain the conclusion.

The critical values that might be required in the testing are provided in the following table:

α	0.001	0.01	0.025	0.05	0.10
$\chi^2_{\alpha,\ \upsilon=3}$	16.26	11.34	9.35	7.82	6.25

where $\chi^2_{\alpha,\ \upsilon=3}$ is determined by the equation $\alpha = \int_{\chi^2_{\alpha,\ \upsilon=3}}^{\infty} \chi^2(x)dx$.

10

7.

A baseline of calibrated length (μ) 200.0m is measured 5 times. Each measurement is independent and made with the same precision. The sample mean ($\bar{x}$) and sample standard deviation (s) are calculated from the measurements:

$\bar{x} = 200.5m$ $\qquad$ $s = 0.05m$

Test at the 95% level of confidence if the measured distance is significantly different from the calibrated distance.

The critical value that might be required in the testing is provided in the following table:

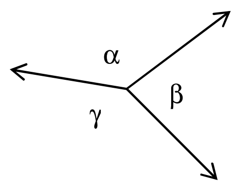
	t_α			
Degree of freedom	$t_{0.90}$	$t_{0.95}$	$t_{0.975}$	$t_{0.99}$
1	3.08	6.31	12.7	31.8
2	1.89	2.92	4.30	6.96
3	1.64	2.35	3.18	4.54

10

	4	1.53	2.13	2.78	3.75	
	5	1.48	2.01	2.57	3.36	

8. Given the angle measurements at a station along with their standard deviations:

Angle	Measurement	Standard Deviation
α	134°38'56"	6.7"
β	83°17'35"	9.9"
γ	142°03'14"	4.3"



Perform least squares adjustment to the problem using

a) Conditional equations (conditional adjustment) 12.5
b) Observation equations (parametric adjustment) 12.5

Total Marks: 100