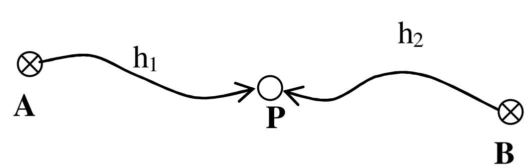
**CANADIAN BOARD OF EXAMINERS FOR PROFESSIONAL SURVEYORS**
**ATLANTIC PROVINCES BOARD OF EXAMINERS FOR LAND SURVEYORS**

**SCHEDULE I / ITEM 2** **October 2007**

# LEAST SQUARES ESTIMATION AND DATA ANALYSIS

**Note: This examination consists of 7 questions on 3 pages.** **Marks**

<table>
<tr><th>Q. No</th><th>Time: 3 hours</th><th>Value</th><th>Earned</th></tr>
<tr><td>1</td><td>Define or explain the following terms:<br>a) Precision<br>b) Accuracy<br>c) Standard deviation<br>d) Root mean square error<br>e) Correlation coefficient<br>f) Redundancy of a linear system<br>g) Type I and type II errors in statistical testing</td><td>15</td><td></td></tr>
<tr><td>2</td><td>Given the following mathematical model<br>$f(\lambda, x) = 0 \quad C_\lambda \quad C_x$<br>where f is the vector of mathematical models, x is the vector of unknown parameters and $C_x$ is its variance matrix, $\lambda$ is the vector of observations and $C_\lambda$ is its variance matrix.<br>a) Linearize the mathematical model<br>b) Formulate the variation function<br>c) Derive the least squares normal equation<br>d) Derive the least squares solution of the unknown parameters.</td><td>15</td><td></td></tr>
<tr><td>3</td><td>Given a leveling network below where A and B are known points, $h_1$ and $h_2$ are two height difference measurements with standard deviation of $\sigma_1$ and $\sigma_2$, respectively and $\sigma_1 = 2\ \sigma_2$. Determine the value of $\sigma_1$ and $\sigma_2$ so that the standard deviation of the height solution at P using least squares adjustment is equal to 2mm.<br><br></td><td>10</td><td></td></tr>
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**4** (10 marks)

Given the variance-covariance matrix of the measurement vector $\lambda = \begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix}$:

$$C_\lambda = \begin{bmatrix} \frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} \end{bmatrix}$$

and two functions of $\lambda$: $x = \lambda_1 + \lambda_2$ and $y = 3\lambda_1$, determine $C_{xy}, C_{x\lambda}, C_{y\lambda}$

**5** (10 marks)

Given the variance-covariance matrix of the horizontal coordinates (x, y) of a survey station, determine the semi-major, semi-minor axis and the orientation of the standard error ellipse associated with this station.

$$C_x = \begin{bmatrix} 0.000532 & 0.000602 \\ 0.000602 & 0.000838 \end{bmatrix} \ m^2$$

**6** (15 marks)

The following residual vector $\hat{r}$ and estimated covariance matrix $C_{\hat{r}}$ were computed from a least squares adjustment using five independent observations with a standard deviation of $\sigma =$ 2 mm and a degree of freedom $\upsilon = 2$:

$$\hat{r} = [4 \quad 2 \quad -3 \quad 10] \qquad \text{(mm)}$$

$$C_{\hat{r}} = \begin{bmatrix} 15 & 1 & 3 & -2 \\ 1 & 7 & -1 & 3 \\ 3 & -1 & 4 & -1 \\ -2 & 3 & -1 & 2 \end{bmatrix} \qquad (\text{mm}^2)$$

Given $\alpha = 0.01$,

a) Conduct a global test to decide if there exists any outlier or not.

b) Conduct local tests to locate possible outlier(s).

The critical values that might be required in the testing are provided in the following tables:

| $\alpha$ | 0.001 | 0.01 | 0.02 | 0.05 | 0.10 |
|---|---|---|---|---|---|
| $\chi^2_{\alpha,\ \upsilon=2}$ | 13.82 | 9.21 | 7.82 | 5.99 | 4.61 |

| $\alpha$ | 0.001 | 0.002 | 0.003 | 0.004 | 0.005 | 0.01 | 0.05 |
|---|---|---|---|---|---|---|---|
| $K_\alpha$ | 3.09 | 2.88 | 2.75 | 2.65 | 2.58 | 2.33 | 1.64 |

where $\chi^2_{\alpha,\ \upsilon=2}$ is determined by the equation $\alpha = \int_{\chi^2_{\alpha,\ \upsilon=2}}^{\infty} \chi^2(x)dx$ and $K_\alpha$ is determined by the equation $\alpha = \int_{K_\alpha}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-x^2/2} dx$.

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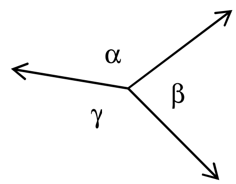
Given the angle measurements at a station along with their standard deviations:

| Angle | Measurement | Standard Deviation |
|---|---|---|
| α | 134°38’56” | 6.7” |
| β | 83°17’35” | 9.9” |
| γ | 142°03’14” | 4.3” |




Perform least squares adjustment to the problem using

a) Conditional equations (conditional adjustment)
b) Observation equations (parametric adjustment)

25

**Total Marks:** 100