CANADIAN BOARD OF EXAMINERS FOR PROFESSIONAL SURVEYORS
ATLANTIC PROVINCES BOARD OF EXAMINERS FOR LAND SURVEYORS

SCHEDULE I / ITEM 2

October 2006

LEAST SQUARES ESTIMATION AND DATA ANALYSIS

Note: This examination consists of 6 questions on 3 pages.

		Marks	
Q. No	Time: 3 hours	Value	Earned
1	Given the following mathematical models $\mathbf{f}_1(\ell_1, \mathbf{x}_1) = 0 \quad \mathbf{C}_{\ell_1} \quad \mathbf{C}_{x_1}$ $\mathbf{f}_2(\ell_2, \mathbf{x}_1, \mathbf{x}_2) = 0 \quad \mathbf{C}_{\ell_2} \quad \mathbf{C}_{x_2}$ where f_1 and f_2 are vectors of mathematical models, x_1 and x_2 are vectors of unknown parameters, ℓ_1 and ℓ_2 are vectors of observations, $C_{\ell_1}, C_{\ell_2}, C_{x_1}$ and C_{x_2} are covariance matrices. a) Linearize the mathematical models (5 marks) b) Formulate the variation function (5 marks) c) Derive the most expanded form of the least squares normal equation system (10 marks)	20	
2	Given the covariance matrix of the horizontal coordinates (x, y) of a survey station, determine the semi-major, semi-minor axis and the orientation of the standard error ellipse associated with this station. $$C_x = \begin{bmatrix} 0.0484 & 0.0246 \\ 0.0246 & 0.0196 \end{bmatrix} \; m^2$$	10	
3	Define or explain the following terms: 1) Type I and type II errors in statistical testing 2) Statistical independence and uncorrelation 3) Expectation 4) Unbiasedness of an estimator 5) Standard deviation 6) Root mean square error 7) Degree of freedom of a linear system 8)	15	

4

A baseline of calibrated length (μ) 1153.00m is measured 5 times. Each measurement is independent and made with the same precision. The sample mean ($\bar{x}$) and sample standard deviation (s) are calculated from the measurements:

$\bar{x}$ =1153.39m   s = 0.06m

a) Describe the major steps to test the mean value. (10 marks)

b) Test at the 90% level of confidence (β) if the measured distance is significantly different from the calibrated distance. (10 marks)

The critical value that might be required in the testing is provided in the following table:

Percentiles of t distribution

	t_β			
Degree of freedom	$t_{0.90}$	$t_{0.95}$	$t_{0.975}$	$t_{0.99}$
1	3.08	6.31	12.7	31.8
2	1.89	2.92	4.30	6.96
3	1.64	2.35	3.18	4.54
4	1.53	2.13	2.78	3.75
5	1.48	2.01	2.57	3.36

20

5

Given the following direct model for the horizontal coordinates (ϕ, λ) of a survey station as a function of ℓ_1 and ℓ_2:

$$\begin{bmatrix} \phi \\ \lambda \end{bmatrix} = \begin{bmatrix} 4 & 5 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} \ell_1 \\ \ell_2 \end{bmatrix}$$

where the covariance matrix of the ℓ's is $C_\ell = \begin{bmatrix} 3 & -1 \\ -1 & 5 \end{bmatrix}$

Compute the covariance matrix for ϕ and λ.

10

6

Given in the following is a diagram of a leveling network to be adjusted.

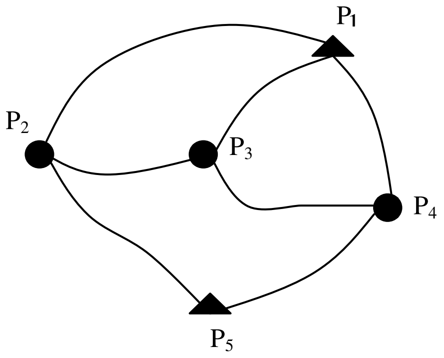

Table 1 contains all of the height difference measurements Δh_{ij} necessary to carry out the adjustment. Assume that all observations have the same standard deviation $\sigma_{\Delta h_{ij}}$ = 0.04m. P1 and P5 are fixed points with known elevations of H_1 = 107.50m and H_5 = 101.00m. Perform least squares adjustment on the leveling network and compute:

a) the adjusted elevations of point 2, 3 and 4 (15 marks);

b) the observation residuals (4 marks);

c) the adjusted height differences (3 marks); and

d) the a-posteriori variance factor $\hat{\sigma}_0$ (3 marks).

Table 1

No.	**i**	**J**	Δh_{ij}
1	P2	P1	1.34
2	P1	P3	-5.00
3	P1	P4	-2.25
4	P2	P3	-3.68
5	P5	P2	5.10
6	P3	P4	2.70
7	P4	P5	-4.13

25

Total Marks: **100**