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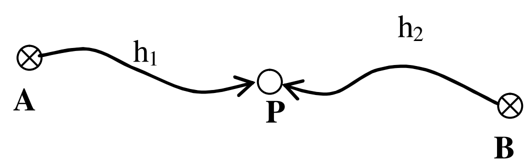
SCHEDULE I / ITEM 2 **October 2005**

LEAST SQUARES ESTIMATION AND DATA ANALYSIS

Note: This examination consists of 7 questions on 3 pages.

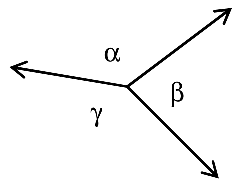
Time: 3 hours

Q. No		Marks Value	Earned
1	Given a leveling network below where A and B are two control points with known height, h_1 and h_2 are two height difference measurements with standard deviation of σ_1 and σ_2, respectively and $\sigma_1 = 2\ \sigma_2$. Determine the value of σ_1 and σ_2 so that the standard deviation of the height solution for point P using least squares adjustment is equal to 2mm. 	10	
2	Given the variance-covariance matrix of the measurement vector $\ell = \begin{bmatrix} \ell_1 \\ \ell_2 \end{bmatrix}$: $C_\ell = \begin{bmatrix} \frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} \end{bmatrix}$ and two functions of ℓ: $x = \ell_1 + \ell_2$ and $y = 3\ell_1$, determine $C_{xy}, C_{x\ell}, C_{y\ell}$	10	
3	Given the variance-covariance matrix of the horizontal coordinates (x, y) of a survey station, determine the semi-major and semi-minor axes and the orientation of the standard error ellipse associated with this station. $C_x = \sigma_0^2 \begin{bmatrix} 0.380 & 0.025 \\ 0.025 & 0.510 \end{bmatrix}$ where $\sigma_0 = 2\text{cm}$.	10	

4 (30 marks)

Given the angle measurements at a station along with their standard deviations:

Angle	Measurement	Standard Deviation
α	134°38’56”	6.7”
β	83°17’35”	9.9”
γ	142°03’14”	4.3”



Perform least squares adjustment to the problem using

a) Conditional equations (conditional adjustment)
b) Observation equations (parametric adjustment)

5 (20 marks)

A baseline of calibrated length (μ) 1153.00m is measured 5 times. Each measurement is independent and made with the same precision. The sample mean ($\bar{x}$) and sample standard deviation (s) are calculated from the measurements:

$\bar{x}$ =1153.39m s = 0.06m

a) Describe the major steps to test the mean value.

b) Test at the 10% level of confidence if the measured distance is significantly different from the calibrated distance.

The critical value that might be required in the testing is provided in the following table:

Percentiles of t distribution

	t_α			
Degree of freedom	$t_{0.90}$	$t_{0.95}$	$t_{0.975}$	$t_{0.99}$
1	3.08	6.31	12.7	31.8
2	1.89	2.92	4.30	6.96
3	1.64	2.35	3.18	4.54
4	1.53	2.13	2.78	3.75
5	1.48	2.01	2.57	3.36

6	Define and explain briefly the following terms: 1) Expectation 2) Unbiasedness of an estimator 3) RMS 4) Null hypothesis and alternative hypothesis 5) Type I error and Type II error	10	
7	Given the following mathematical models $f_1(\ell_1, x_1) = 0 \quad C_{\ell_1}$ $f_2(\ell_2, x_1, x_2) = 0 \quad C_{\ell_2} C_{x_2}$ where f_i, x_i, ℓ_i and C_i represent mathematical model vectors, unknown parameter vectors, observation vectors and covariance matrices. a) Linearize the mathematical models b) Formulate the variation function.	10	
	Total Marks:	100	