CANADIAN BOARD OF EXAMINERS FOR PROFESSIONAL SURVEYORS

C-3 ADVANCED SURVEYING — March 2012

Note: This examination consists of 6 questions and formulae on 7 pages.

Although programmable calculators may be used, candidates must show all formulae used, the substitution of values into them, and any intermediate values to 2 more significant figures than warranted for the answer. Otherwise, full marks may not be awarded even though the answer is numerically correct.

Q. No	Time: 3 hours	Marks Value	Marks Earned
1.	Often, provincial or other authorities require that measurements, s_{ij}, by an EODMI or total station be done on a calibration baseline that has known pillar coordinates, $\mathbf{x} = [x_1, x_2, \ldots, x_n]^T$ with $\mathbf{C_x}$. Many of the calibration baselines were established at least 25 years ago when EODMI were ± 5 mm ± 5 ppm, compared to ± 1 mm ± 2 ppm commonly encountered today. Nonetheless, some aspects of EODMI behaviour can be investigated by using an *ad hoc* collinear array of points such as a series of tribrachs on tripods. ISO standard 17123-4 requires an array of 7 points with spacing following the Heerbrugg design, as explained by Rüeger in his *Introduction to EDM*. The spacing is based on the unit length [$U = \lambda_{mod}/2$] of the EODMI and on the overall length of the array, which is usually at least as long as any intended use of the EODMI. In either the baseline or the array configuration, the redundancy allows for a least squares estimation. Compare the use of a 7 point collinear array to the use of a 7 pillar calibration baseline under each of the following considerations. a) [1] Information that is known *a priori*; b) [1] What setting out must be done in preparation for measurements by the subject EODMI; c) [1] What quantities are "observed"; d) [2] What corrections are applied as "pre-processing" [i.e., before the estimation] and why; e) [4] What quantities are estimated and a typical observation equation with an explanation of the variables; f) [5] The algorithm, in matrix notation with dimensioning, for the estimation, with an explanation of the elements in a typical row of each design matrix; g) [3] What statistical testing can be done *a posteriori* [null and alternate hypotheses, statistic, test]; h) [1] How the results are used in subsequent employment of the EODMI. i) [2] The advantages and disadvantages of using a 7 point collinear array rather than a 7 point calibration baseline.	20	

2.	A local plane coordinate system was established at the collar of a shaft at a latitude of 59°N. At a depth of 2 km, an adit runs approximately in an easterly direction. A flat traverse follows the adit with stations along one side. Gyro-azimuths, following the transit method, have been observed at regular intervals in order to "control" the orientation of the adit. The transit method results in an angle, A_g, describing the direction of the gyro zero with respect to North. The equipment and procedures suggest that $\sigma_{Ag} = \pm 5''$. Explain the corrections, with suggestions of their values, which should be applied to an A_g, observed at 4 km from the shaft in order to convert it to a grid azimuth in the surface coordinate system. If you are not able to calculate a value for a correction, explain what other information would be needed to do so and how it would be obtained.	10	
3.	Consider that a total station is setup over station A with a height of instrument HI. A reflector is setup over point B, with a height of HR. The elevation of station A, H_A, is known ± 2 mm. The elevation of point B, H_B, is to be determined. One set has resulted in the zenith angle, z_{AB}, being 55° ± 3" and the slope distance, s_{AB}, being 220 m ± 2 mm. a) Explain, and show in a sketch, a compatible method for measuring HI and HR and the values of σ_{HI} and σ_{HR}. b) Using plane trigonometry and the method of a), determine the uncertainty in H_B. c) Explain how additional sets would improve the uncertainty in H_B and what would have to be altered before each set in order for that improvement. d) Suggest, with some substantiation [i.e., calculation], a limit on the length of sight that would allow the use of plane trigonometry [i.e., beyond which the geodetic aspects would have to be regarded].	10 10 5 5	
4.	A campaign of observations, at t_1, can be adjusted to estimate the coordinates of the points involved, based on $\mathbf{l}_1 + \mathbf{v}_1 = \mathbf{A}_1\mathbf{x}_1$ ["**l**" etc. (bold) denote vectors, "**A**" (bold) denotes a matrix]. During a later campaign, at t_2, the observations can be repeated so that $\mathbf{l}_2 + \mathbf{v}_2 = \mathbf{A}_2\mathbf{x}_2$. If there are object points on a sensitive structure, its behaviour can be described geometrically with respect to the reference points, using the displacement field resulting from $\mathbf{d}_x = \mathbf{x}_2 - \mathbf{x}_1$, in which some of the elements of the $\mathbf{d}_x$ vector are for the object points, say $\mathbf{d}_{x\ obj}$. It may be possible to difference the observations, $\mathbf{d}_l = \mathbf{l}_2 - \mathbf{l}_1$, so that the displacement field can be estimated, based on $\mathbf{d}_l + \mathbf{v}_d = \mathbf{A}\mathbf{d}_x$. a) Explain the conditions under which the $\mathbf{d}_x$ can be calculated in the coordinate differencing approach, with respect to the $\mathbf{l}_i$, $\mathbf{A}_i$ and $\mathbf{x}_i$, and the advantages and disadvantages of this approach. b) Explain the conditions under which the observation differencing approach may be followed and its advantages and disadvantages. c) Explain which approach can accommodate geotechnical data and give an example of an appropriate geotechnical observable, with its observation equation and an explanation of how the value recorded in the field becomes the observation value. d) If the monitoring were to endure over a long period of time, say several decades, explain what concerns would arise in each of the two approaches and how best to deal with those concerns.	1 1 2 1	

5.	An abandoned railway, running in a North/South direction, has a right of way [RoW] width of 300 m. Although the track has been removed, some useful structures remain. A topographic plan of a 5 km portion of the RoW is to be produced with a contour interval of 0.2 m and at a hardcopy scale of 1:500 and some detail, with spot heights, at 1:200. Since the railway runs through a heavily developed urban area, there is no at-grade access but there are overpasses at about every 500 m in this 5 km portion. There is a survey marker at the middle of each overpass, in the centre of the sidewalk along the North side of the overpass. Successive markers are intervisible. Each marker has coordinates [E_i,N_i] in the municipal mapping plane. Previous surveys have resulted in the markers having station ellipses with major semi-axes of 0.08 m at 95%. Their elevations [H_i] are, at best, ±0.30 m at 95%. With substantiation, suggest an approach to gathering the data necessary to produce the plan. The suggestion should include: a) placement of coordinated points [existing or additional or both]: i.e., points to be used as control [horizontal and vertical] and occupied points for the data gathering [sketch, approximately to scale]; b) technique [instrumentation specifics, procedures] to be used in coordinating the points [illustrated in a separate sketch]; c) technique [instrumentation specifics, procedures] to be used in the data gathering [illustrated in a separate sketch]; d) quality assurance and quality control [QA/QC] measures during data gathering; e) QA/QC during processing; f) creation of the final product. If you decide that the data gathering would be similar in several sub-portions of the 5 km, explain that and, after explaining the placement of coordinated points [a) and b)], then explain what would be done in a typical sub-portion [c) through f)]. The a) through f) sub-sectioning of this question are to facilitate the organization of your answer. The assessment will have regard for the overall approach and will not be broken down into subsections.	30	
6.	A flat hanging traverse is to be measured with uniform sight lengths of 100 m ±2 mm. There are two "fixed" stations, "A" and "B", plus seven traverse stations, "P1" to "P7" so that "B" and "P1" to "P6" would be occupied while "A" and "P7" would be sighted. One approach is to measure the included horizontal angles [values near 180°] with σ_β = ±5". An alternative method is to occupy certain stations and to observe the azimuth to the next station using a gyro attachment so that σ_A = ±15". a) [3] If only included angles were observed, explain the dominant influence on the random positional uncertainty at the end point of the traverse, "P7", and suggest a value and orientation of the uncertainty. b) [2] If azimuths rather than included angles were observed, explain the dominant influence on the random positional uncertainty at the end point of the traverse, "P7", and suggest a value and orientation of the uncertainty.	5	
	Total Marks:	100	

Percentiles of the χ^2 distribution:

	0.50	0.70	0.80	0.90	0.95	0.975	0.99	0.995
1	0.455	1.07	1.64	2.71	3.84	5.02	6.63	7.88
2	1.39	2.41	3.22	4.61	5.99	7.38	9.21	10.60
3	2.37	3.66	4.64	6.25	7.81	9.35	11.34	12.84

Some potentially useful formulae are given below.

$$\sqrt{\sigma_c^2} \approx \pm 0.001h; \sqrt{\sigma_c^2} = \pm 0.0005h; \sqrt{\sigma_c^2} \le \pm 0.0005h; \sqrt{\sigma_c^2} \le \pm 0.0001$$

$$\sigma_{\delta_c}^2 = \frac{\sigma_{c_F}^2 + \sigma_{c_T}^2}{s_{FT}^2}$$

$$\sigma_{\beta_C}^2 = \frac{\sigma_{c_F}^2}{s_F^2} + \frac{\sigma_{c_T}^2}{s_T^2} + \left[\frac{1}{s_F^2} + \frac{1}{s_T^2} - \frac{2}{s_F s_T}\cos\beta\right]\sigma_{c_A}^2$$

$$\sigma_l = \pm 0.2div; \ \sigma_l = \pm 0.02div; \ \sigma_l \le \pm 0.5"$$

$$\sigma_{\beta_l} = \pm \sigma_l \sqrt{\cot^2 z_i + \cot^2 z_j}$$

$$\pm \frac{30"}{M} \le \sigma_p \le \pm \frac{60"}{M}$$

$$\sigma_{ps} \approx \frac{70"}{M}$$

$$b = 2a + c$$

$$a = \frac{120}{206264.8}\frac{D}{M}$$

$$2" \le c \le 4"$$

$$\sigma_r \ge \pm 0.3div; \sigma_r = \pm 0.3div; \sigma_r = \pm 2.5div; \sigma_r = \pm 0.6"$$

$$\sigma_z^2 = \sigma_{z_l}^2 + \sigma_{z_p}^2 + \sigma_{z_r}^2$$

$$\sigma_{z_l} = \pm \sigma_l$$

$$\sigma_{z_p} = \pm \frac{\sigma_p}{\sqrt{2}}$$

$$\sigma_{z_r} = \pm \frac{\sigma_r}{\sqrt{2}}$$

$$\sin\beta_1 = \frac{b_1 \sin\alpha_1}{a}; \quad \sin\beta_2 = \frac{b_2 \sin\alpha_2}{a}$$

$$\sigma_\beta^2 = \frac{\tan^2\beta}{b^2}\sigma_b^2 + \frac{\tan^2\beta}{a^2}\sigma_a^2 + \left(\frac{b^2}{a^2\cos^2\beta} - \tan^2\beta\right)\sigma_\alpha^2$$

$$\sigma_{y_n}^2 = \sum_{i=1}^{n-1}(x_n - x_i)^2\sigma_{\beta_i}^2; \quad \sigma_{y_n}^2 = \sum_{i=1}^{n-1}(x_{i+1} - x_i)^2\sigma_{\alpha_i}^2$$

$$\sigma_s^2 = a^2 + b^2 s^2$$

$$d\delta = 8'' \frac{pS}{T^2}\frac{dT}{dx}$$

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1 atm = 1013.25 mb = 101.325 kPa = 760 torr = 760 mmHg
0 C = 273.15 K
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$$T = \frac{\sum_{i=1}^{n}\left[\left(h_{i+1} - h_i\right)\left(T_i + T_{i+1}\right)\right]}{2\left(h_n - h_1\right)}$$

$$\Delta h_w = \frac{w}{aE}\left(Lh - \frac{h^2}{2}\right)$$

$$n_a = 1 + \frac{0.359474(0.0002945)p}{273.15 + t}$$

$$n_a = 1 + \frac{0.359474(0.0002821)p}{273.15 + t}$$

$$\Delta N_1 = 294.5 - \frac{0.29065p}{1 + 0.00366086t}$$

$$\Delta N_1 = 282.1 - \frac{0.29065p}{1 + 0.00366086t}$$

$$\varepsilon_A = \frac{206264.8}{b}\sqrt{e_1^2 + e_2^2}$$

$$e_i^2 = \left[\frac{e}{2}\right]^2 + [2r]^2 + [0.2mm]^2$$

$$\Delta H = \frac{PH}{aE}; \qquad E = 2.1 \text{x} 10^6 \text{ kgcm}^{-2}$$

$$T = 2\pi\sqrt{\frac{H}{g}}; \qquad g = 980 \text{ cms}^{-2}$$

$$e = \frac{30hHdv^2}{P}$$

$$r_0 = r_2 - \frac{P_1\left(r_1 - r_2\right)}{P_2 - P_1}$$

$$r = \frac{\pi d^4 E}{64RP}$$

$N = N' + \Delta N$; $\Delta N = ca\Delta t$; $E = A - A_g = t \pm \gamma - A_g$

$$\theta = \frac{d \tan\phi \left(1 - \varepsilon^2 \sin^2\phi\right)^{\frac{1}{2}}}{a}$$

$$\Delta\gamma = \frac{\Delta E \tan\phi}{R}$$

6378206.4 m, 0.0822718948;   6378137.0 m, 0.081819191

$$\varepsilon = \frac{\Delta\ell}{\ell}$$

$$d_x = r_{x_1} - r_{x_2}; \quad d_y = r_{y_1} - r_{y_2}$$

$$\theta_x = \frac{d_x}{s}; \quad \theta_y = \frac{d_y}{s}$$

$$c = [N_0 - N_a]s$$

$$c_{cal} = \frac{s_{std} - s_{obs}}{s_{std}} s_i; \; c_{align} = -\frac{d^2}{2s}; \; c_{temp} = \alpha(t - t_0)s; \; c_{tens} = \frac{P - P_0}{aE} s$$

$$c_{sag} = -\frac{s^3}{24}\left(\frac{mg\cos\theta}{P}\right)^2\left(1 \pm \frac{mgs\sin\theta}{P}\right); \quad c_{sea} = \frac{H}{R+H} s$$

$$\frac{s^2}{\sigma^2} \le \frac{1}{\nu}\chi^2_{\nu,1-\alpha}?; \quad \frac{1}{F_{\nu_1,\nu_2,1-\frac{\alpha}{2}}} \le \frac{s_1^2}{s_2^2} \le F_{\nu_1,\nu_2,1-\frac{\alpha}{2}}?; \quad \frac{a_\mu}{s_{a_\mu}} \le t_{\nu,1-\frac{\alpha}{2}}?$$

$$C_x = \sigma_0^2\left[C_{x_S}^{-1} + \left(A^T P A\right)_U\right]^{-1}$$

$$\Delta_{f/b} \le \pm 3mm\sqrt{K}; \; \Delta_{f/b} \le \pm 4mm\sqrt{K}; \; \Delta_{f/b} \le \pm 8mm\sqrt{K}; \; \Delta_{f/b} \le \pm 24mm\sqrt{K}$$

$d_{y1} = r_{1,1} - r_{2,1}$; $d_{y2} = r_{1,2} - r_{2,2}$; $\Delta y = d_{y2} - d_{y1}$

$$T = \frac{\Delta y}{\Delta H}$$

$$s_{ij} + z_0 = x_j - x_i; ks_{ij} + z_0 = x_j - x_i; s = s' + s'\Delta N$$

$c+r = 0.0675\ K^2$

$$A = iU; \quad B_0 = \frac{1}{15}[C_0 - 6A - U]; \quad D = \frac{U}{36}$$

$$1to2: A + 1B + 3D$$
$$2to3: A + 3B + 7D$$
$$3to4: A + 5B + 11D$$
$$4to5: A + 4B + 9D$$
$$5to6: A + 2B + 5D$$
$$6to7: A + D$$

$$d_4 = 2R \arcsin \sqrt{\frac{R^2 \sin^2 (d_1 \frac{k}{2R}) - k^2 \frac{(H_2 - H_1)^2}{4}}{k^2 (R + H_1)(R + H_2)}}$$

$$d_4 = R \arctan \left[\frac{d_2 \sin(z_1 + \varepsilon_1 + \delta)}{R + H_1 + d_2 \cos(z_1 + \varepsilon_1 + \delta)} \right]$$