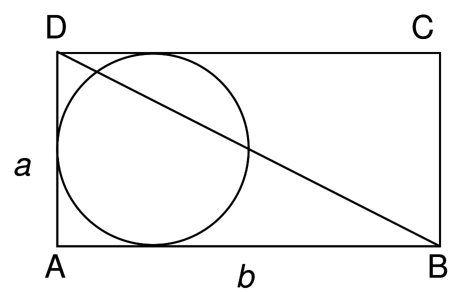
CANADIAN BOARD OF EXAMINERS FOR PROFESSIONAL SURVEYORS

C-2 LEAST SQUARES ESTIMATION & DATA ANALYSIS — March 2012

Although programmable calculators may be used, candidates must show all formulae used, the substitution of values into them, and any intermediate values to 2 more significant figures than warranted by the answer. Otherwise, full marks may not be awarded even though the answer is numerically correct.

Note: This examination consists of 8 questions on 3 pages.

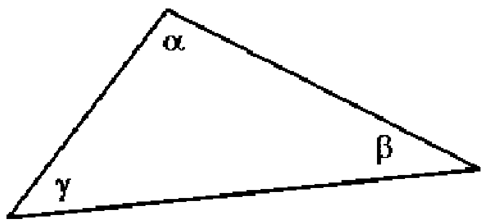
Time: 3 hours

Q. No		Marks Value	Marks Earned
1.	Briefly explain the following terms a) Precision b) Accuracy c) Internal reliability d) External reliability e) Correlation coefficient	10	
2.	Given the following over-determined linear system: y = Ax C_y where y is the vector of observations and C_y is its variance-covariance matrix, x is the vector of unknown parameters, A is the design matrix. a) Derive the least squares normal equation b) Derive the least squares solution of the unknown parameters.	10	
3.	Sides *a* and *b* are measured once each as follows: $\boldsymbol{l} = \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 10 \\ 20 \end{bmatrix}$ m $\boldsymbol{C}_l = \begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix}$ cm^2  a) Estimate the areas of triangle ABD and the circle shown inside the rectangle. b) Estimate the standard deviations of the quantities computed in part a). c) Estimate the correlation between the triangle and the circle estimates. d) Discuss the nature of the correlations computed in part c).	15	
4.	Prove that $\frac{\sigma}{\sqrt{n}}$ is the standard deviation of the mean value $\bar{x} = \frac{\sum_{i=1}^{n} \ell_i}{n}$ and each measurement ℓ_i is made with a standard deviation σ.	10	

5.

Given the angle measurements of a triangle along with their standard deviations:

Angle	Measurement	Standard Deviation
α	104°38'56"	6.7"
β	33°17'35"	9.9"
γ	42°03'14"	4.3"



Perform least squares adjustment to the problem using

a) Conditional equations (conditional adjustment) 12.5

b) Observation equations (parametric adjustment) 12.5

6.

Given the variance-covariance matrix of the horizontal coordinates (x, y) of a survey station, determine the semi-major, semi-minor axis and the orientation of the standard error ellipse associated with this station. 10

$$C_x = \begin{bmatrix} 0.000532 & 0.000602 \\ 0.000602 & 0.000838 \end{bmatrix} \ m^2$$

7.

Given the sample unit variance obtained from the adjustment of a geodetic network $\hat{\sigma}_0^2 = 0.55\,cm^2$ with a degree of freedom $\upsilon = 3$ and the a-priori standard deviation $\sigma_0 = 0.44$ cm , conduct a statistic test to decide if the adjustment result is acceptable with a significance level of $\alpha = 5\%$. Provide the major test steps and explain the conclusion. 10

The critical values that might be required in the testing are provided in the following table:

α	0.001	0.01	0.025	0.05	0.10
$\chi^2_{\alpha,\ \upsilon=3}$	16.26	11.34	9.35	7.82	6.25

where $\chi^2_{\alpha,\ \upsilon=3}$ is determined by the equation $\alpha = \int_{\chi^2_{\alpha,\ \upsilon=3}}^{\infty} \chi^2(x)dx$.

8.	A baseline of calibrated length (μ) 200.0m is measured 5 times. Each measurement is independent and made with the same precision. The sample mean ($\bar{x}$) and sample standard deviation (s) are calculated from the measurements: $\bar{x}$ = 200.5m s = 0.05m Test at the 95% level of confidence if the measured distance is significantly different from the calibrated distance. The critical value that might be required in the testing is provided in the following table:	10	
	Total Marks:	100	

	t_α			
Degree of freedom	$t_{0.90}$	$t_{0.95}$	$t_{0.975}$	$t_{0.99}$
1	3.08	6.31	12.7	31.8
2	1.89	2.92	4.30	6.96
3	1.64	2.35	3.18	4.54
4	1.53	2.13	2.78	3.75
5	1.48	2.01	2.57	3.36