CANADIAN BOARD OF EXAMINERS FOR PROFESSIONAL SURVEYORS

C-3 ADVANCED SURVEYING **March 2011**

Note: This examination consists of 6 questions and formulae on 7 pages.

Although programmable calculators may be used, candidates must show all formulae used, the substitution of values into them, and any intermediate values to 2 more significant figures than warranted for the answer. Otherwise, full marks may not be awarded even though the answer is numerically correct.

Time: 3 hours

Q. No		Marks Value	Marks Earned
1.	A campaign of observations, at t_1, can be adjusted to estimate the coordinates of the points involved, based on $\mathbf{l}_1 + \mathbf{v}_1 = \mathbf{A}_1\mathbf{x}_1$ ["$\mathbf{l}$" etc. (bold) denote vectors, "$\mathbf{A}$" (bold) denotes a matrix]. During a later campaign, at t_2, the observations can be repeated so that $\mathbf{l}_2 + \mathbf{v}_2 = \mathbf{A}_2\mathbf{x}_2$. If there are object points on a sensitive structure, its behaviour can be described geometrically with respect to the reference points, using the displacement field resulting from $\mathbf{d}_x = \mathbf{x}_2 - \mathbf{x}_1$, in which some of the elements of the $\mathbf{d}_x$ vector are for the object points, say $\mathbf{d}_{x\ obj}$. It may be possible to difference the observations, $\mathbf{d}_l = \mathbf{l}_2 - \mathbf{l}_1$, so that the displacement field can be estimated, based on $\mathbf{d}_l + \mathbf{v}_d = \mathbf{A}\mathbf{d}_x$.		
	a) Explain the conditions under which the $\mathbf{d}_x$ can be calculated in the coordinate differencing approach, with respect to the $\mathbf{l}_i$, $\mathbf{A}_i$ and $\mathbf{x}_i$, and the advantages and disadvantages of this approach.	3	
	b) Explain the conditions under which the observation differencing approach may be followed and its advantages and disadvantages.	3	
	c) Explain which approach can accommodate geotechnical data and give an example of an appropriate geotechnical observable, with its observation equation and an explanation of how the value recorded in the field becomes the observation value.	6	
	d) If the monitoring were to endure over a long period of time, say several decades, explain what concerns would arise in each of the two approaches and how best to deal with those concerns.	5	
	e) With reference to sketches [plan and profile views], explain the two types of relative movement that can be derived from repeated plumbline table readings at two elevations, say $r_{1,i}$ at H_1 and $r_{2,i}$ at H_2, with $H_2 < H_1$ ["2" is below "1"] and the readings at time t_i. The explanation should also address the algebraic signs of the movement and of the tilt.	8	
2.	A local plane coordinate system was established at the collar of a shaft at a latitude of 57°N. At a depth of 2 km, an adit runs approximately in an easterly direction. A flat traverse follows the adit with stations along one side. Gyro-azimuths, following the transit method, have been observed at regular intervals in order to "control" the orientation of the adit. The transit method results in an angle, A_g, describing the direction of the gyro zero with respect to North. The equipment and procedures suggest that $\sigma_{Ag} = \pm 5''$. Explain the corrections, with suggestions of their values, that should be applied to an A_g, observed at 3 km from the shaft in order to convert it to a grid azimuth in the surface coordinate system. If you are not able to calculate a value for a correction, explain what other information would be needed to do so and how it would be obtained.	10	

3.	a) A Canada-wide system of elevations is provided by "Vertical Control" benchmarks established by the Geodetic Survey of Canada. Explain the concept behind orders of vertical control and how the orders relate to benchmark spacing and the quality of an elevation difference between any pair of the same order. b) A construction project within a block [300 m by 300 m] requires control of high precision, $\Delta H \pm 1$ mm or better. There are two Canadian Special Order benchmarks at diagonally opposite corners of the project which are accessible only along the outside of the block, along the street [so maximum length of a route of levelling would be 300 m to 600 m (from the centre of the block and along one or two sides) to one of the benchmarks]. Any established elevation would be the result of averaging a forward and backward levelling. With numerical substantiation, explain whether the two Special Order benchmarks would be suitable. If they are not sufficient, explain what you would recommend as suitable control.	5 5	
4.	Often, provincial or other authorities require that measurements, s_{ij}, by an EODMI or total station be done on a calibration baseline that has known pillar coordinates, $\mathbf{x} = [x_1, x_2, \ldots, x_n]^T$ with $\mathbf{C_x}$. Many of the calibration baselines were established at least 25 years ago when EODMI were ± 5 mm ± 5 ppm, compared to ± 1 mm ± 2 ppm commonly encountered today. However, some aspects of EODMI behaviour can be investigated by using an *ad hoc* collinear array of points such as a series of tribrachs on tripods. ISO standard 17123-4 requires an array of 7 points with spacing following the Heerbrugg design, as explained by Rüeger in his *Introduction to EDM*. The spacing is based on the unit length [$U = \lambda_{mod}/2$] of the EODMI and on the overall length of the array, which is usually at least as long as any intended use of the EODMI. Compare the use of a linear array to the use of a calibration baseline under each of the following considerations. a) Information that is known *a priori*; b) What setting out must be done in preparation for measurements by the subject EODMI. c) What is "observed"; d) What corrections are applied as "pre-processing" and why; e) What quantities are estimated and a typical observation equation with an explanation of the variables; f) What statistical testing can be done *a posteriori* [null and alternate hypotheses, statistic, test]; g) How the results are used in subsequent employment of the EODMI. h) The advantages and disadvantages of the linear array rather than the baseline.	1 1 2 2 3 2 1 3	

5.	Consider that a total station is setup over station A with a height of instrument HI. A reflector is setup over point B, with a height of HR. The elevation of station A, H_A, is known ± 2 mm. The elevation of point B, H_B, is to be determined. One set has resulted in the zenith angle, z_{AB}, being 55° ± 3" and the slope distance, s_{AB}, being 200 m ± 2 mm. a) Suggest a compatible method for measuring HI and HR and the values of σ_{HI} and σ_{HR}. b) Using plane trigonometry and the method of a), determine the uncertainty in H_B. c) Explain how additional sets would improve the uncertainty in H_B and what would have to be altered before each set in order for that improvement. d) Suggest, with some substantiation [i.e., calculation], a limit on the length of sight that would allow the use of plane trigonometry [i.e., beyond which the geodetic aspects would have to be regarded].	5 10 5 5	
6.	Observations on α *Ursae Minoris*, might be done at a latitude of about 55° in many locations in Canada. As with any other angular measurement, averaging the determinations, by the hour angle method, from several sets improves the precision of the azimuth. The accuracy of the determination would also be improved with several sets. One set results in the angle from the star to the RO, the value of which is the average of face left and face right circle readings. Assume that an instrument comparable in precision to a Wild T2 [micrometer: 1"; 28X; plate vial: 20"/div; vertical circle index vial: 30" with coincidence viewing] were being used and that the RO is at least 250 m away and at about the same elevation as the occupied station and that the line is almost due east-west. a) With regard for the ability to center, level, point and read, suggest a value for the standard deviation of the angle, from the star to the RO, in one set. b) Explain what systematic influences would affect a single determination, or set, and how the accuracy would be improved with several sets. c) Explain how your answers to a) and b) would change if a comparable precision electronic theodolite with a biaxial compensator were used rather than the T2.	5 5 5	
	Total Marks:	100	

Percentiles of the χ^2 distribution:

	0.50	0.70	0.80	0.90	0.95	0.975	0.99	0.995
1	0.455	1.07	1.64	2.71	3.84	5.02	6.63	7.88
2	1.39	2.41	3.22	4.61	5.99	7.38	9.21	10.60
3	2.37	3.66	4.64	6.25	7.81	9.35	11.34	12.84

Some potentially useful formulae are given below.

$$\sqrt{\sigma_c^2} \approx \pm 0.001h$$

$$\sqrt{\sigma_c^2} = \pm 0.0005h$$

$$\sqrt{\sigma_c^2} \leq \pm 0.0005h$$

$$\sqrt{\sigma_c^2} \leq \pm 0.0001$$

$$\sigma_{\delta_c}^2 = \frac{\sigma_{c_F}^2 + \sigma_{c_T}^2}{s_{FT}^2}$$

$$\sigma_{\beta_C}^2 = \frac{\sigma_{c_F}^2}{s_F^2} + \frac{\sigma_{c_T}^2}{s_T^2} + \left[\frac{1}{s_F^2} + \frac{1}{s_T^2} - \frac{2}{s_F s_T}\cos\beta\right]\sigma_{c_A}^2$$

$$\sigma_l = \pm 0.2 div$$

$$\sigma_l = \pm 0.02 div$$

$$\sigma_l \le \pm 0.5''$$

$$\sigma_{\beta_l} = \pm\sigma_l\sqrt{\cot^2 z_i + \cot^2 z_j}$$

$$\pm\frac{30''}{M} \le \sigma_p \le \pm\frac{60''}{M}$$

$$\sigma_{ps} \approx \frac{70''}{M}$$

$$b = 2a + c$$

$$a = \frac{120}{206264.8}\frac{D}{M}$$

$$2'' \le c \le 4''$$

$$\sigma_r \ge \pm 0.3 div$$

$$\sigma_r = \pm 0.3 div$$

$$\sigma_r = \pm 2.5 div$$

$$\sigma_r = \pm 0.6''$$

$$\sigma_z^2 = \sigma_{z_l}^2 + \sigma_{z_p}^2 + \sigma_{z_r}^2$$

$$\sigma_{z_l} = \pm\sigma_l$$

$$\sigma_{z_p} = \pm\frac{\sigma_p}{\sqrt{2}}$$

$$\sigma_{z_r} = \pm\frac{\sigma_r}{\sqrt{2}}$$

$$\sin\beta_1 = \frac{b_1\sin\alpha_1}{a};\sin\beta_2 = \frac{b_2\sin\alpha_2}{a}$$

$$\sigma_\beta^2 = \frac{\tan^2\beta}{b^2}\sigma_b^2 + \frac{\tan^2\beta}{a^2}\sigma_a^2 + \left(\frac{b^2}{a^2\cos^2\beta} - \tan^2\beta\right)\sigma_\alpha^2$$

$$\sigma_{y_n}^2 = \sum_{i=1}^{n-1}(x_n - x_i)^2\sigma_{\beta_i}^2$$

$$\sigma_{y_n}^2 = \sum_{i=1}^{n-1} (x_{i+1} - x_i)^2 \sigma_{\alpha_i}^2$$

$$\sigma_s^2 = a^2 + b^2 s^2$$

$$d\delta = 8'' \frac{pS}{T^2} \frac{dT}{dx}$$

1 atm = 1013.25 mb = 101.325 kPa = 760 torr = 760 mmHg
0 C = 273.15 K

$$T = \frac{\sum_{i=1}^{n} \left[(h_{i+1} - h_i)(T_i + T_{i+1}) \right]}{2(h_n - h_1)}$$

$$\Delta h_w = \frac{w}{aE} \left(Lh - \frac{h^2}{2} \right)$$

$$n_a = 1 + \frac{0.359474\,(0.0002945\,)\,p}{273.15 + t}$$

$$n_a = 1 + \frac{0.359474\,(0.0002821)\,p}{273.15 + t}$$

$$\Delta N_1 = 294.5 - \frac{0.29065\,p}{1 + 0.00366086\,t}$$

$$\Delta N_1 = 282.1 - \frac{0.29065\,p}{1 + 0.00366086\,t}$$

$$\varepsilon_A = \frac{206264.8}{b} \sqrt{e_1^2 + e_2^2}$$

$$e_i^2 = \left[\frac{e}{2} \right]^2 + [2r]^2 + [0.2mm]^2$$

$$\Delta H = \frac{PH}{aE}; \qquad \mathrm{E} = 2.1\mathrm{x}10^6 \ \mathrm{kgcm}^{-2}$$

$$T = 2\pi \sqrt{\frac{H}{g}}; \qquad \mathrm{g} = 980 \ \mathrm{cms}^{-2}$$

$$e = \frac{30hHdv^2}{P}$$

$$r_0 = r_2 - \frac{P_1(r_1 - r_2)}{P_2 - P_1}$$

$$r = \frac{\pi d^4 E}{64RP}$$

N = N' + ΔN; ΔN = caΔt; E = A - A_g = t ± γ - A_g

$$\theta = \frac{d \tan\phi \left(1 - \varepsilon^2 \sin^2 \phi\right)^{\frac{1}{2}}}{a}$$

$$\Delta\gamma = \frac{\Delta E \tan\phi}{R}$$

6378206.4 m, 0.0822718948;   6378137.0 m, 0.081819191

$$\varepsilon = \frac{\Delta\ell}{\ell}$$

$$d_x = r_{x_1} - r_{x_2}; d_y = r_{y_1} - r_{y_2}$$

$$\theta_x = \frac{d_x}{s}; \theta_y = \frac{d_y}{s}$$

$$c = [N_0 - N_a]s$$

$$c_{cal} = \frac{s_{std} - s_{obs}}{s_{std}} s_i$$

$$c_{align} = -\frac{d^2}{2s}$$

$$c_{temp} = \alpha(t - t_0)s$$

$$c_{tens} = \frac{P - P_0}{aE} s$$

$$c_{sag} = -\frac{s^3}{24}\left(\frac{mg\cos\theta}{P}\right)^2\left(1 \pm \frac{mgs\sin\theta}{P}\right)$$

$$c_{sea} = \frac{H}{R+H}s$$

$$\frac{s^2}{\sigma^2} \le \frac{1}{\nu}\chi^2_{\nu,1-\alpha}\ ?$$

$$\frac{1}{F_{\nu_1,\nu_2,1-\frac{\alpha}{2}}} \le \frac{s_1^2}{s_2^2} \le F_{\nu_1,\nu_2,1-\frac{\alpha}{2}}\ ?$$

$$\frac{a_\mu}{s_{a_\mu}} \le t_{\nu,1-\frac{\alpha}{2}}\ ?$$

$$C_x = \sigma_0^2\left[C_{x_S}^{-1} + \left(A^T PA\right)_U\right]^{-1}$$

$$\Delta_{f/b} \le \pm 3mm\sqrt{K}$$

$$\Delta_{f/b} \le \pm 4mm\sqrt{K}$$

$$\Delta_{f/b} \le \pm 8mm\sqrt{K}$$

$$\Delta_{f/b} \le \pm 24mm\sqrt{K}$$

$d_{y1} = r_{1,1} - r_{2,1}$; $d_{y2} = r_{1,2} - r_{2,2}$; $\Delta y = d_{y2} - d_{y1}$

$$T = \frac{\Delta y}{\Delta H}$$