CANADIAN BOARD OF EXAMINERS FOR PROFESSIONAL SURVEYORS
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SCHEDULE I / ITEM 2

March 1007

LEAST SQUARES ESTIMATION AND DATA ANALYSIS

Note: This examination consists of 8 questions on 3 pages. **Marks**

Time: 3 hours

Q. No		Value	Earned
1	Given the following mathematical models $\mathbf{f}_1(\ell_1, \mathbf{x}_1) = 0 \quad \mathbf{C}_{\ell_1} \quad \mathbf{C}_{x_1}$ $\mathbf{f}_2(\ell_2, \mathbf{x}_1, \mathbf{x}_2) = 0 \quad \mathbf{C}_{\ell_2} \quad \mathbf{C}_{x_2}$ where f_1 and f_2 are vectors of mathematical models, x_1 and x_2 are vectors of unknown parameters, ℓ_1 and ℓ_2 are vectors of observations, $C_{\ell_1}, C_{\ell_2}, C_{x_1}$ and C_{x_2} are covariance matrices. a) Linearize the mathematical models b) Formulate the variation function	10	
2	Given n independent measurements of a single quantity: $\ell_1, \ell_2, ..., \ell_n$ and their corresponding standard deviations: $\sigma_1, \sigma_2, ..., \sigma_n$, derive the expression for the variance of the mean value using the Law of Propagation of Variances.	10	
3	Given the following variance-covariance matrix from a least squares adjustment for the coordinates of point A and B, $C_x = \begin{bmatrix} 10 & 5 & 2 & 1 \\ 5 & 10 & 1 & 1 \\ 2 & 1 & 5 & 2 \\ 1 & 1 & 2 & 5 \end{bmatrix} m^2$ where $x = [x_A \quad y_A \quad x_B \quad y_B]^T$. a) Compute the correlation coefficient between x_A and y_B. b) Compute the correlation coefficient between y_A and x_B. c) Compute the variance-covariance matrix for the coordinate differences $\Delta x = x_B - x_A$ and $\Delta y = y_B - y_A$.	10	

<table>
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<td>4</td>
<td>Given the covariance matrix of the horizontal coordinates (x, y) of a survey station, determine the semi-major, semi-minor axis and the orientation of the standard error ellipse associated with this station.

$$C_x = \begin{bmatrix} 0.0484 & 0.0246 \\ 0.0246 & 0.0196 \end{bmatrix} m^2$$</td>
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<td>5</td>
<td>Define or explain the following terms:

1) Precision
2) Accuracy
3) Statistical independence
4) Type I error
5) Type II error
6) Expectation
7) Degree of freedom of a linear system</td>
<td>15</td>
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<td>6</td>
<td>Given 20 independent measurements of a distance made with the same standard deviation of 0.033m and the calculated mean value $\mu = 37.615$m, construct a 95% confidence interval for the population mean.

Percentiles of Standard Normal Distribution

<table>
<tr><td>α</td><td>0.002</td><td>0.003</td><td>0.004</td><td>0.005</td><td>0.01</td><td>0.025</td><td>0.05</td></tr>
<tr><td>K_α</td><td>2.88</td><td>2.75</td><td>2.65</td><td>2.58</td><td>2.33</td><td>1.96</td><td>1.64</td></tr>
</table>

where K_α is determined by the equation $\alpha = \int_{K_\alpha}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-x^2/2} dx$.</td>
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<td>A baseline of calibrated length (μ) 1153.00m is measured 5 times independently with the same precision. The sample mean ($\bar{x}$) and sample standard deviation (s) are calculated from the measurements:

$\bar{x}$ =1153.39m s = 0.06m

Test at a 90% confidence level (α) if the measured distance is significantly different from the calibrated distance.

Percentiles of t distribution

<table>
<tr><td></td><td colspan="4">t_α</td></tr>
<tr><td>Degree of freedom</td><td>$t_{0.90}$</td><td>$t_{0.95}$</td><td>$t_{0.975}$</td><td>$t_{0.99}$</td></tr>
<tr><td>1</td><td>3.08</td><td>6.31</td><td>12.7</td><td>31.8</td></tr>
<tr><td>2</td><td>1.89</td><td>2.92</td><td>4.30</td><td>6.96</td></tr>
<tr><td>3</td><td>1.64</td><td>2.35</td><td>3.18</td><td>4.54</td></tr>
<tr><td>4</td><td>1.53</td><td>2.13</td><td>2.78</td><td>3.75</td></tr>
<tr><td>5</td><td>1.48</td><td>2.01</td><td>2.57</td><td>3.36</td></tr>
</table></td>
<td>10</td>
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8

Given in the following is a diagram of a leveling network to be adjusted.

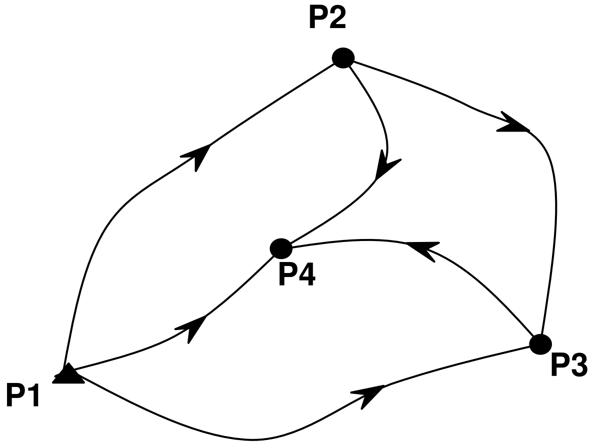

Table 1 contains all of the height difference measurements Δh_{ij} necessary to carry out the adjustment. Assume that all observations have the same standard deviation $\sigma_{\Delta h_{ij}}$ = 0.04m. P1 is a fixed point with known elevations of H_1 = 101.00m. Perform least squares adjustment on the leveling network to compute:

a) the adjusted elevations of point P2, P3 and P4 and their variance-covariance matrix;

b) the observation residuals;

c) the adjusted height differences; and

d) the a-posteriori variance factor $\hat{\sigma}_0$.

Table 1

No.	**i**	**j**	Δh_{ij} (unit: metre)
1	P1	P3	6.15
2	P1	P4	12.54
3	P3	P4	6.46
4	P1	P2	1.11
5	P2	P4	11.55
6	P2	P3	5.10

25

Total Marks: 100