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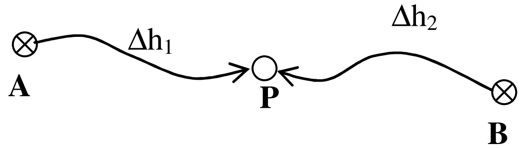
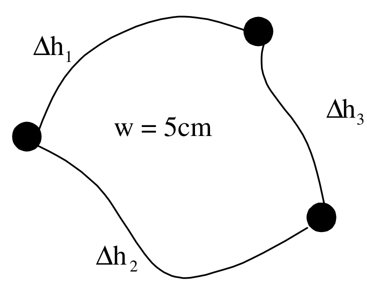
SCHEDULE I / ITEM 2 **March 2006**

LEAST SQUARES ESTIMATION AND DATA ANALYSIS

Note: This examination consists of 6 questions on 3 pages. **Marks**

Time: 3 hours

Q. No		Value	Earned
1	Define and explain briefly the following terms: a) Precision and accuracy b) Statistical independence and uncorrelation c) Null hypothesis and alternative hypothesis d) Redundancy of a linear system e) Unbiasedness of an estimator	10	
2	Given the following mathematical models $f_1(\ell_1, x_1, x_2) = 0 \quad C_{\ell_1}\ C_{x_1}$ $f_2(\ell_2, x_2) = 0 \quad C_{\ell_2}$ where f_i, x_i, ℓ_i and C_i represent mathematical model vectors, unknown parameter vectors, observation vectors and covariance matrices. a) Linearize the mathematical models b) Formulate the variation function c) Derive the most expanded form of the least squares normal equation system.	20	
3	Given a leveling network below where A and B are two control points with known heights, Δh_1 and Δh_2 are two height difference measurements with standard deviations of σ_1 and σ_2, respectively and $\sigma_1 = 0.5\ \sigma_2$. Determine the value of σ_1 and σ_2 so that the standard deviation of the height solution for point P using least squares adjustment is equal to 4 mm.  	10	

4	Given the following direct model for the horizontal coordinates (x, y) of a survey station as a function of ℓ_1, ℓ_2 and ℓ_3: $$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 & -2 & 1 \\ 5 & 3 & -1 \end{bmatrix} \begin{bmatrix} \ell_1 \\ \ell_2 \\ \ell_3 \end{bmatrix}$$ where the covariance matrix of the ℓ's is $C_\ell = \begin{bmatrix} 4 & -2 & -1 \\ -2 & 5 & 1 \\ -1 & 1 & 2 \end{bmatrix}$ a) Compute the covariance matrix for x and y. b) Determine the semi-major, semi-minor axis and the orientation of the standard error ellipse associated with this station.	20	
5	Perform a least squares adjustment of the following leveling network in which three height differences Δh_i (i = 1, 2, 3) were observed with a variance of 2 cm^2. The misclosure w is 5 cm. . 	20	

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The following residual vector $\hat{r}$ and estimated covariance matrix $C_{\hat{r}}$ were computed from a least squares adjustment using five independent observations with a standard deviation of $\sigma = 2$ mm and a degrees of freedom $\upsilon = 2$:

$$\hat{r} = \begin{bmatrix} 4 & 2 & -3 & 9 & -5 \end{bmatrix} \quad \text{(mm)}$$

$$C_{\hat{r}} = \begin{bmatrix} 9 & -1 & 2 & -3 & 5 \\ -1 & 1 & 1 & 3 & -2 \\ 2 & 1 & 4 & -2 & 3 \\ -3 & 3 & -2 & 4 & -1 \\ 5 & -2 & 3 & -1 & 16 \end{bmatrix} \quad (\text{mm}^2)$$

a) Describe the major steps of statistical testing for outlier detection and identification.

b) Conduct a global test to decide if there exists any outlier in the measurements with a confidence level of $1 - \alpha = 99\%$.

c) If the test in a) fails, conduct local tests to locate the outlier(s).

The critical values that might be required in the testing are provided in the following tables:

α	0.001	0.01	0.02	0.05	0.10
$\chi^2_{\alpha,\ \upsilon=2}$	13.82	9.21	7.82	5.99	4.61

α	0.001	0.002	0.003	0.004	0.005	0.01	0.05
K_α	3.09	2.88	2.75	2.65	2.58	2.33	1.64

where $\chi^2_{\alpha,\ \upsilon=2}$ is determined by the equation $\alpha = \int_{\chi^2_{\alpha,\ \upsilon=2}}^{\infty} \chi^2(x)dx$ and K_α is determined by the equation $\alpha = \int_{K_\alpha}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-x^2/2} dx$.

20

Total Marks: **100**