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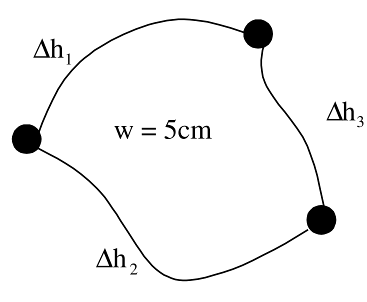
SCHEDULE I / ITEM 2 **March 2004**

LEAST SQUARES ESTIMATION AND DATA ANALYSIS

Note: This examination consists of _6_ questions on _3_ pages.

Time: 3 hours

Q. No		Marks Value	Marks Earned
1	Define and explain briefly the following terms: a) Expectation b) Variance c) Unbiasedness of an estimator d) Mean square error e) RMS f) Null hypothesis and alternative hypothesis g) Type I error and Type II error h) Statistical independence and uncorrelation	20	
2	Given the variance-covariance matrix of the horizontal coordinates (x, y) of a survey station, determine the semi-major, semi-minor axis and the orientation of the standard error ellipse associated with this station. $C_x = \begin{bmatrix} 0.0484 & 0.0246 \\ 0.0246 & 0.0196 \end{bmatrix} \ m^2$	10	
3	Perform a least squares adjustment of the following leveling network in which three height differences Δh_i, i = 1, 2, 3 were observed.  The misclosure w is 5cm. Each Δh_i was measured with a variance of 3 cm^2.	20	

4 (20 marks)

The following residual vector $\hat{r}$ and estimated covariance matrix $C_{\hat{r}}$ were computed from a least squares adjustment using five independent observations with a standard deviation of $\sigma = 2$ mm and a degree of freedom $\upsilon = 2$:

$$\hat{r} = \begin{bmatrix} 4 & 2 & -3 & 10 \end{bmatrix} \qquad \text{(mm)}$$

$$C_{\hat{r}} = \begin{bmatrix} 15 & 1 & 3 & -2 \\ 1 & 7 & -1 & 3 \\ 3 & -1 & 4 & -1 \\ -2 & 3 & -1 & 2 \end{bmatrix} \qquad (\text{mm}^2)$$

Given $\alpha = 0.01$,

a) Conduct a global test to decide if there exists any outlier or not.

b) If the test in a) fails, conduct local tests to locate the outlier(s).

The critical values that might be required in the testing are provided in the following tables:

α	0.001	0.01	0.02	0.05	0.10
$\chi^2_{\alpha,\ \upsilon=2}$	13.82	9.21	7.82	5.99	4.61

α	0.001	0.002	0.003	0.004	0.005	0.01	0.05
K_α	3.09	2.88	2.75	2.65	2.58	2.33	1.64

where $\chi^2_{\alpha,\ \upsilon=2}$ is determined by the equation $\alpha = \int_{\chi^2_{\alpha,\ \upsilon=2}}^{\infty} \chi^2(x)dx$ and

K_α is determined by the equation $\alpha = \int_{K_\alpha}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-x^2/2} dx$.

5 (5 marks)

Given the following direct model for the horizontal coordinates (ϕ, λ) of a survey station as a function of λ_1 and λ_2:

$$\begin{bmatrix} \phi \\ \lambda \end{bmatrix} = \begin{bmatrix} 4 & 5 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix}$$

where the covariance matrix of the λ's is $C_\lambda = \begin{bmatrix} 3 & -1 \\ -1 & 5 \end{bmatrix}$

Compute the covariance matrix for ϕ and λ.

6	Given the following mathematical models $f_1(\lambda_1, x_1, x_2) = 0 \quad C_{\lambda_1} \quad C_{x_1}$ $f_2(\lambda_2, x_1, x_2) = 0 \quad C_{\lambda_2} \quad C_{x_2}$ where f_1 and f_2 are vectors of mathematical models, x_1 and x_2 are vectors of unknown parameters, λ_1 and λ_2 are vectors of observations, $C_{\lambda_1}, C_{\lambda_2}, C_{x_1}$ and C_{x_2} are covariance matrices. a) Formulate the variation function. b) Derive the most expanded form of the least squares normal equation system.	25	
	Total Marks:	100	