ASSOCIATION OF CANADA LANDS SURVEYORS - BOARD OF EXAMINERS
WESTERN CANADIAN BOARD OF EXAMINERS FOR LAND SURVEYORS
ATLANTIC PROVINCES BOARD OF EXAMINERS FOR LAND SURVEYORS

SCHEDULE I / ITEM 2 **March 2003**

LEAST SQUARES ESTIMATION AND DATA ANALYSIS

Note: This examination consists of _5_ questions on _2_ pages. **Marks**

Time: 3 hours

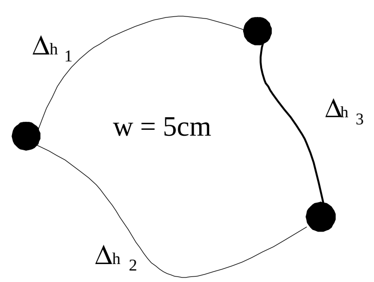
<table>
<tr><th>Q. No</th><th></th><th>Value</th><th>Earned</th></tr>
<tr><td>1</td><td>

Define and explain briefly the following terms:

a) Type I and II errors in statistical testing
b) Degree of freedom of a linear system
c) Accuracy
d) Precision
e) Unbiasedness of an estimator

</td><td>10</td><td></td></tr>
<tr><td>2</td><td>

Perform a least squares adjustment of the following leveling network in which three height differences Δh_i, i = 1, 2, 3 were observed.



The misclosure w is 5cm. Each Δh_i was measured with a variance of 2 cm^2.

</td><td>20</td><td></td></tr>
<tr><td>3</td><td>

Given the following direct model for the horizontal coordinates (x, y) of a survey station x and y as a function of ℓ_1, ℓ_2 and ℓ_3:

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 & -2 & 1 \\ 5 & 3 & -1 \end{bmatrix} \begin{bmatrix} \ell_1 \\ \ell_2 \\ \ell_3 \end{bmatrix}$$

where the covariance matrix of the ℓ's is $C_\ell = \begin{bmatrix} 4 & -2 & -1 \\ -2 & 2 & 1 \\ -1 & 1 & 2 \end{bmatrix}$

a) Compute the covariance matrix for x and y.
b) Determine the semi-major, semi-minor axis and the orientation of the standard error ellipse associated with this station.

</td><td>20</td><td></td></tr>
</table>

4

Given the following mathematical models

$$f_1(\ell_1, x_1, x_2) = 0 \qquad C_{\ell_1} \quad C_{x_1} \quad C_{x_2}$$
$$f_2(\ell_2, x_1, x_2) = 0 \qquad C_{\ell_2}$$

where f_1 and f_2 are vectors of mathematical models, x_1 and x_2 are vectors of unknown parameters, ℓ_1 and ℓ_2 are vectors of observations, $C_{\ell_1}, C_{\ell_2}, C_{x_1}$ and C_{x_2} are covariance matrices.

a) Formulate the variation function.

b) Derive the most expanded form of the least squares normal equation system.

25

5

The following residual vector $\hat{r}$ and estimated covariance matrix $C_{\hat{r}}$ were computed from a least squares adjustment using five independent observations with a standard deviation of s = 2mm and a degree of freedom $u = 2$:

$$\hat{r} = [4 \quad 2 \quad -3 \quad 9 \quad -5] \qquad \text{(mm)}$$

$$C_{\hat{r}} = \begin{bmatrix} 9 & -1 & 2 & -3 & 5 \\ -1 & 1 & 1 & 3 & -2 \\ 2 & 1 & 4 & -2 & 3 \\ -3 & 3 & -2 & 4 & -1 \\ 5 & -2 & 3 & -1 & 16 \end{bmatrix} \qquad (\text{mm}^2)$$

Given α = 0.01,

a) Conduct a global test to decide if there exists any outlier or not.

b) If the test in a) fails, conduct local tests to locate the outlier(s).

The critical values that might be required in the testing are provided in the following tables:

α	0.001	0.01	0.02	0.05	0.10
$c^2_{a,\,u=2}$	13.82	9.21	7.82	5.99	4.61

α	0.001	0.002	0.003	0.004	0.005	0.01	0.05
K_α	3.09	2.88	2.75	2.65	2.58	2.33	1.64

where $c^2_{a,\,u=2}$ is determined by the equation $a = \int_{c^2_{a,\,u=2}}^{\infty} c^2(x)dx$ and

K_α is determined by the equation $\alpha = \int_{K_\alpha}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-x^2/2} dx$.

25

Total Marks: 100 0