CANADIAN BOARD OF EXAMINERS FOR PROFESSIONAL SURVEYORS

SCHEDULE I / ITEM 2
LEAST SQUARES ESTIMATION & DATA ANALYSIS

March 2009

Although programmable calculators may be used, candidates must show all formulae used, the substitution of values into them, and any intermediate values to 2 more significant figures than warranted by the answer. Otherwise, full marks may not be awarded even though the answer is numerically correct.

Note: This examination consists of 7 questions on 3 pages. **Marks**

Q. No Time: 3 hours Value Earned

1. Define and briefly explain the following terms

   a) Precision
   b) Accuracy
   c) Root mean square error
   d) Correlation coefficient
   e) Redundancy of a linear system

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2. Given the following mathematical models

$$f(\lambda, x) = 0 \qquad C_\lambda \quad C_x$$

where f is the vector of the mathematical model, x is the vector of unknown parameters and C_x is its variance matrix, λ is the vector of observations and C_λ is its variance matrix.

a) Provide the linearized form of the given mathematical model.

b) Formulate the variation function.

c) Derive the least squares solution of the unknown parameters.

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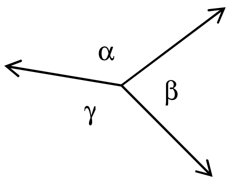
3. Given the angle measurements at a station along with their standard deviations:

Angle	Measurement	Standard Deviation
α	134°38'56"	6.7"
β	83°17'35"	9.9"
γ	142°03'14"	4.3"



Apply the least squares adjustment to the problem using

a) Conditional equations (conditional adjustment)
b) Observation equations (parametric adjustment)

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4.

Given the variance-covariance matrix of the horizontal coordinates (x, y) of a survey station, determine the semi-major, semi-minor axis and the orientation of the standard error ellipse associated with this station.

$$C_x = \begin{bmatrix} 0.000532 & 0.000602 \\ 0.000602 & 0.000838 \end{bmatrix} \ m^2$$

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5.

Given two distance measurements that are independent and have standard deviations σ_1 = 0.20m and σ_2 = 0.15m, respectively,

a) Calculate the standard deviations of the sum and difference of the two measurements.

b) Calculate the correlation between the sum and the difference.

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6.

The following residual vector $\hat{r}$ and estimated covariance matrix $C_{\hat{r}}$ were computed from a least squares adjustment using five independent observations with a standard deviation of $\sigma = 2$ mm and a degree of freedom $\upsilon = 2$:

$$\hat{r} = \begin{bmatrix} 4 & 2 & -3 & 10 \end{bmatrix} \qquad \text{(mm)}$$

$$C_{\hat{r}} = \begin{bmatrix} 15 & 1 & 3 & -2 \\ 1 & 7 & -1 & 3 \\ 3 & -1 & 4 & -1 \\ -2 & 3 & -1 & 2 \end{bmatrix} \qquad (\text{mm}^2)$$

Given $\alpha = 0.01$,

a) Conduct a global test to decide if there exists any outlier or not.

b) Conduct local tests to locate possible outlier(s).

The critical values that might be required in the testing are provided in the following tables:

α	0.001	0.01	0.02	0.05	0.10
$\chi^2_{\alpha,\ \upsilon=2}$	13.82	9.21	7.82	5.99	4.61

α	0.001	0.002	0.003	0.004	0.005	0.01	0.05
K_α	3.09	2.88	2.75	2.65	2.58	2.33	1.64

where $\chi^2_{\alpha,\ \upsilon=2}$ is determined by the equation $\alpha = \int_{\chi^2_{\alpha,\ \upsilon=2}}^{\infty} \chi^2(x)dx$ and K_α is determined by the equation $\alpha = \int_{K_\alpha}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-x^2/2} dx$.

15

7.

A baseline of calibrated length (μ) 200.0m is measured 5 times. Each measurement is independent and made with the same precision. The sample mean ($\overline{x}$) and sample standard deviation (s) are calculated from the measurements:

$\overline{x}$ = 200.5m   s = 0.05m

Test at the 95% level of confidence if the measured distance is significantly different from the calibrated distance. The critical value that might be required in the testing is provided in the following table:

Percentiles of t distribution

	t_α			
Degree of freedom	$t_{0.90}$	$t_{0.95}$	$t_{0.975}$	$t_{0.99}$
1	3.08	6.31	12.7	31.8
2	1.89	2.92	4.30	6.96
3	1.64	2.35	3.18	4.54
4	1.53	2.13	2.78	3.75
5	1.48	2.01	2.57	3.36

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Total Marks: 100