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SCHEDULE 1 / ITEM _2
LEAST SQUARES ESTIMATION AND DATA ANALYSIS

February 2000
(1990 Regulations)
(Closed Book)

Time: 3 hours **Marks**

Note: This examination consists of 5 questions on 2 pages

PROBLEM 1 **15**

Given below is the covariance matrix obtained from a least-squares adjustment for a survey station:

$$\mathbf{C_p} = \begin{bmatrix} 0.0484 & 0.0246 \\ 0.0246 & 0.0196 \end{bmatrix} \text{ (metre}^2\text{)}$$

Calculate the semi-major axis, semi-minor axis, and the orientation of the standard error ellipse associated with this position error.

PROBLEM 2 **25**

Given the following mathematical models

$$\mathbf{f}_1(\ell_1, \mathbf{x}_1) = 0 \quad \mathbf{C}_{\ell_1} \quad \mathbf{C}_{x_1}$$
$$\mathbf{f}_2(\ell_2, \mathbf{x}_2) = 0 \quad \mathbf{C}_{\ell_2}$$
$$\mathbf{f}_3(\mathbf{x}_2) = 0 \quad \mathbf{C}_{f_3}$$

where $\mathbf{f}_1, \mathbf{f}_2$ *and* $\mathbf{f}_3$ are vectors of mathematical models, $\mathbf{x}_1$ *and* $\mathbf{x}_2$ are vectors of unknown parameter, ℓ_1 *and* ℓ_2 are vectors of observations, $\mathbf{C}_{\ell_1}, \mathbf{C}_{\ell_2}, \mathbf{C}_{x_1}$ *and* $\mathbf{C}_{f_3}$ are covariance matrices.

a) Formulate the variation function.

Derive the most expanded form of the least squares normal equation system

PROBLEM 3 **15**

Define and explain briefly the following terms:

a) Type I error
b) Type II error
c) Accuracy
d) Precision
e) Filtering

PROBLEM 4 **15**

Given the following direct model for y_1 and y_2 as a function of x_1, x_2 and x_3:

$$y_1 = 5x_1 - 2x_2 + 3x_3 + 9$$

$$y_2 = 3x_1 - x_2 + 2x_3 + 4$$

where $x_1 = x_2 = x_3 = 1$ and the covariance matrix of the x's: $\mathbf{C}_x = \begin{bmatrix} 3 & -1 & -2 \\ -1 & 5 & 1 \\ -2 & 1 & 4 \end{bmatrix}$

Compute the covariance matrix $\mathbf{C}_y$ for y's.

PROBLEM 5 **30**

Given a plane triangle in which the three angles have been observed:

$a = 40°00'15''$ $\quad\quad$ $b = 85°00'10''$ $\quad\quad$ $g = 54°59'15''$

All angles were measured with equal precision ($s_a^2 = s_b^2 = s_g^2 = 16 \ arc\sec^2$) and assumed to be uncorrelated. Perform a least-squares adjustment and

a) Compute the solution vector **x** and its covariance matrix $\mathbf{C}_{\hat{\mathbf{x}}}$.

b) Compute the residual vector **r** and its covariance matrix $\mathbf{C}_{\hat{\mathbf{r}}}$.

c) Compute the variance factor $\hat{s}_o^2$.

d) Perform a test on the estimated variance factor at significance level $a = 0.05$.

e) Perform a test for gross errors on each estimated residual at significance level $a = 0.01$.

The critical values that might be required in the testing are provided in the following tables:

a	K_a
0.001	3.09
0.002	2.88
0.003	2.75
0.004	2.65
0.005	2.58
0.01	
0.05	

2.33
1.64

a
0.001
0.005
0.01
0.02
0.05
0.10

$c^{2}_{a,v}(v=1)$

10.83

7.88

6.63

5.41

3.84

2.71

$c^{2}_{a,v}(v=2)$

13.82

10.60

9.21

7.82

5.99

4.61

where K_a is determined by the equation $a = \int_{K_a}^{\infty} \frac{1}{\sqrt{2p}} e^{-x^2/2} dx$ and $c^{2}_{a,v}$ is determined by the equation $a = \int_{c_{a,v}}^{\infty} f(c^2) dc^2$ in which v is the degrees of freedom.